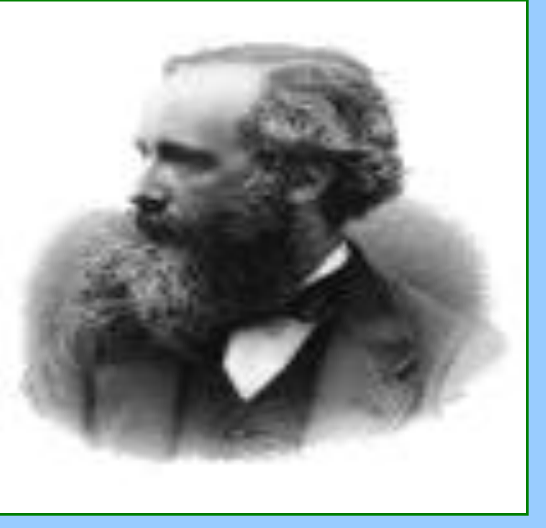
# LECTURE 1

# ELECTROMAGNETISM



MAXWELL

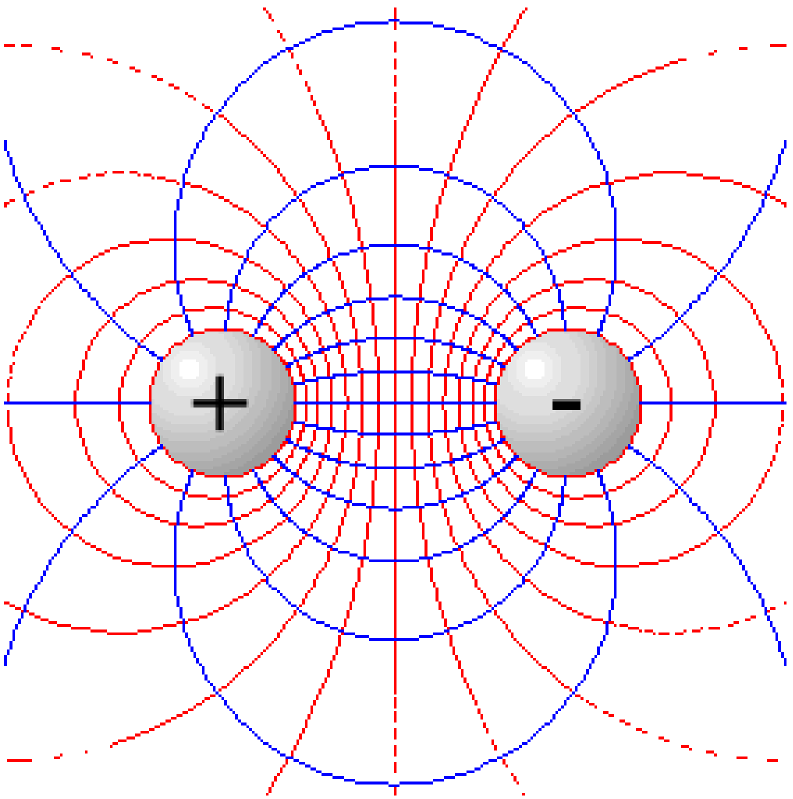


G NAGENDRA PRASAD

- Electromagnetism – Introduction,
- Electrostatics, Coulombs inverse square law,
- Electric field and  Electric Potential,
- Electric flux and Gauss Law
- Magnetostatics, Magnetic Dipole, Magnetic flux,
- Magnetic field intensity, Relation between $\mu_r$ and $\chi$,
- Bohr magneton and current density

# Introduction to Electromagnetism

- Electromagnetism is a branch of Physics that describes the interactions involving electric charge.
- This includes the phenomena of Electricity, Magnetism, Electromagnetic induction (Electric generators) and Electromagnetic radiation.



# Electrostatics

- **Electrostatics is the branch of Physics, which deals with the behavior of stationary electric charges.**

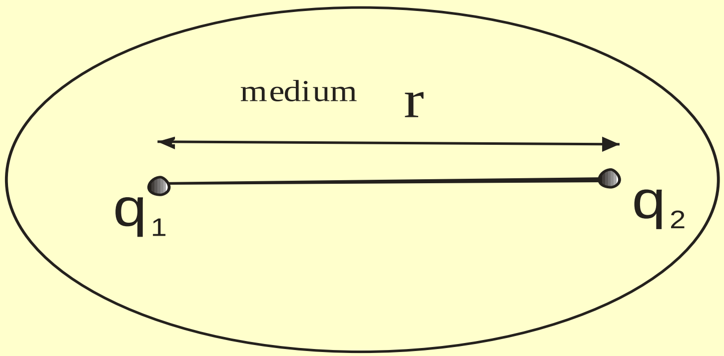
- **Charges are existing in two different kinds called positive and negative, these charges when in combination add algebraically i.e. the charge is a scalar quantity always quantized in integral multiples of electronic charge.**

- **Charge is a fundamental property of the ultimate particles making up matter, the total charge of a closed system cannot change i.e. net charge is conserved in an isolated system**

# Basic definitions

## Coulomb’s Inverse Square Law

**Coulomb’s inverse square law gives the force between the two charges. According to this law, the force (F) between two electrostatic point charges ($q_1$ and $q_2$) is proportional to the product of the charges and inversely proportional to the square of the distance (r) separating the *charges.***




$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2} \qquad F = K\frac{q_1 q_2}{r^2}$$

**where K is proportionality constant which depends on the nature of the medium.**
**This force acts along the line joining the charges. For a dielectric medium of relative permittivity $\varepsilon_r$ ,the value of K is given by,**

$$K = \frac{1}{4\pi\varepsilon_0\varepsilon_r} = \frac{1}{4\pi\varepsilon}$$

**where $\varepsilon$ = permittivity of the medium.**

# Electric field

- Electric charges affect the space around them.

- The space around the charge within which its effect is felt or experienced is called Electric field.

- Electric field Intensity (or) Strength of the Electric field, due to a point charge $q_a$ at a given point is defined as the force per unit charge exerted on a test charge $q_b$ placed at that point in the field.

$$\overrightarrow{E_a} = \frac{\overrightarrow{F_{ba}}}{q_b} = \frac{q_a \quad \hat{r}_a}{4\pi\varepsilon_0 \, r^2} \qquad \text{volt m}^{-1} \ \text{(or)} \ \text{N C}^{-1}$$

# Electrostatic Potential (V)

➢ **The Electric potential is defined as the amount of work done in moving unit positive charge from infinity to the given point of the field of the given charge against the electrical force.**

➢ **Unit: volt (or) joule / coulomb**

Potential $$V = -\int_{\infty}^{r} E \,.\, dx = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 x^2}\, dx$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{q}{4\pi\varepsilon_0 r}$$

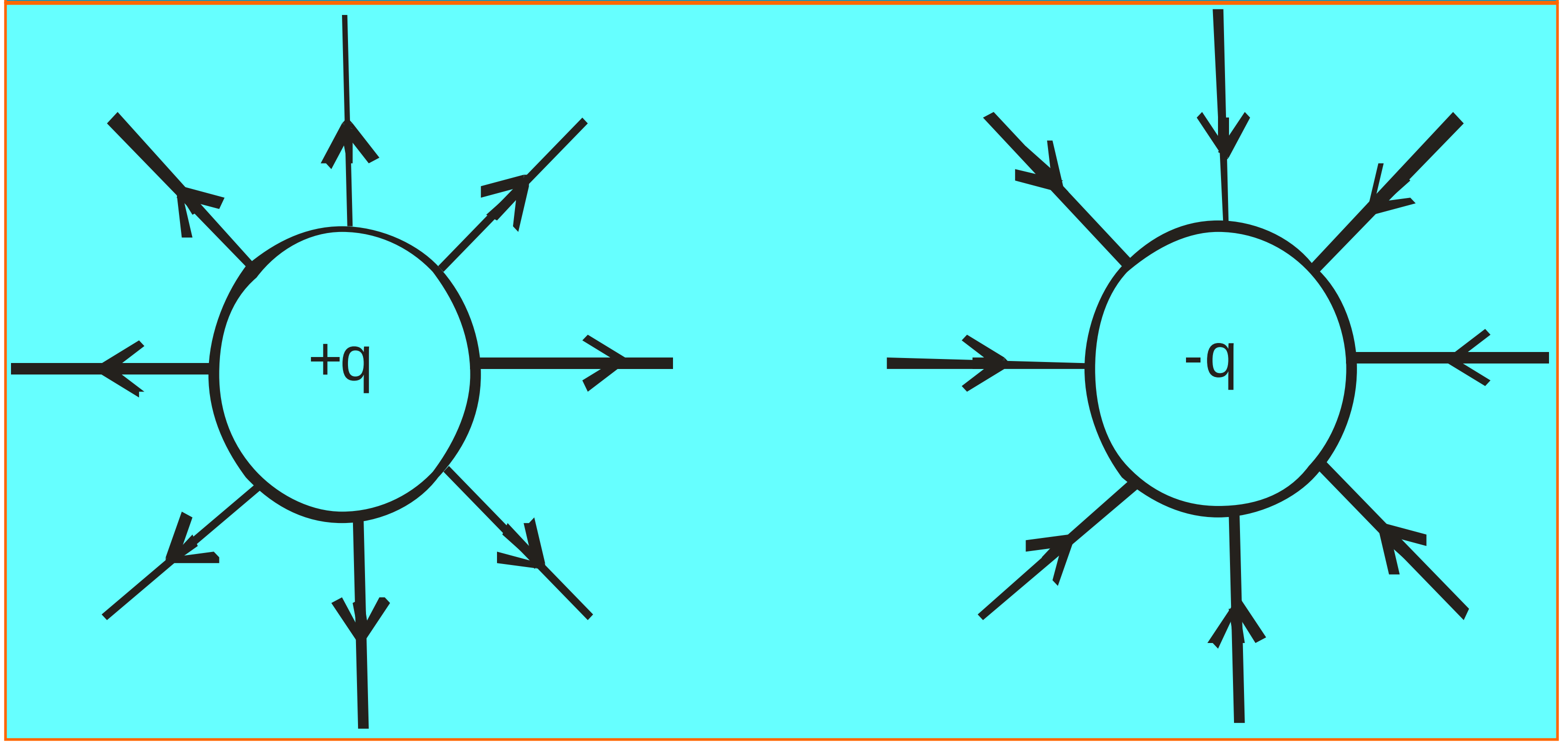
# Electric lines of force

- An Electric field may be described in terms of lines of force in much the same way as a magnetic field.

**Properties of electric lines of force**

1. Every Lines of force originates from a positive charge and terminates on negative charge.
2. Lines of force never intersect.
3. The tangent to Lines of force at any point gives the direction of the electric field E at that point.
4. The number of Lines of force per unit area at right angles to the lines is proportional to the magnitude of E.
5. Each unit positive charge gives rise to lines of force in free space.

## Representation of electric lines of force for Isolated positive and negative charges.



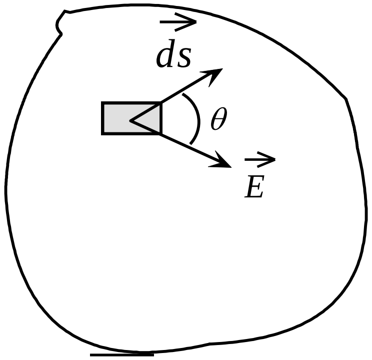
# Electric flux

- **The Electric flux is defined as the number of lines of force that pass through a surface placed in the electric field.**

- **The Electric flux ($d\varphi$) through elementary area ds is defined as the product of the area and the component of electric field strength normal to the area.**

# Electric flux expression

The electric flux normal to the area $ds$ = $d\varphi = \vec{E}.\ \vec{ds}$




$d\varphi = E\ ds \cos\theta = (E \cos\theta)\ .\ ds$

= (*Component of E* along the direction of the normal × *area)*

**Flux of the electric field**

*The flux over the entire surface* $= \varphi = \oint_S d\phi$

$$= \oint_S E \cos\theta\ .\ ds$$

**Unit: $Nm^2\ C^{-1}$**

# Gauss theorem (or) Gauss law

- **This Law relates the flux through any closed surface and the net charge enclosed within the surface.**

- **The Electric flux ($\varphi$) through a closed surface is equal to the $1/\varepsilon_0$ times the net charge $q$ enclosed by the surface.**

$$\phi = \left(\frac{1}{\varepsilon_0}\right) q$$

(or)

$$\phi = \left(\frac{q}{\varepsilon_0}\right) = \oint E\, ds \cos\theta$$

# Electric flux density (or) Electric displacement vector (D)

- **It is defined as the number of Electric Lines of force passing normally through an unit area of cross section in the field. It is given by,**

$$D = \frac{\phi}{A}$$

**Unit : Coulomb / $m^2$**

# Permittivity (μ)

Permittivity is defined as the ratio of electric displacement vector (*D*) in a dielectric medium to the applied electric field strength (*E*).

$$\varepsilon = \frac{D}{E}$$

- Mathematically it is given by,

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

- **$\varepsilon_0$ = permittivity of free space or vacuum**
- **$\varepsilon_r$ = permittivity or dielectric constant of the medium**

**Unit: Farad /metre**

# Magnetostatics

- ***Magnetostatics*** deals with the behaviour of stationary Magnetic fields.
- Oersterd and Ampere proved experimentally that the current carrying conductor produces a magnetic field around it.
- The origin of Magnetism is linked with current and magnetic quantities are measured in terms of current.

## Magnetic dipole

- Any two opposite magnetic poles separated by a distance *d* constitute a magnetic dipole.

# Magnetic dipole moment ($\mu_m$)

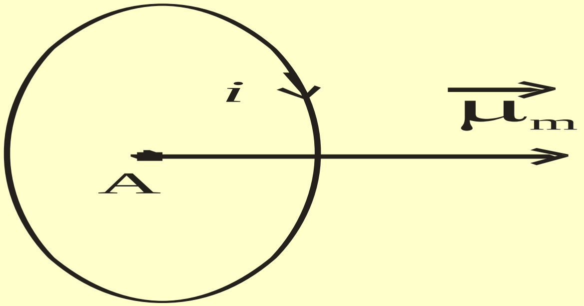
- If *m* is the magnetic pole strength and *l* is the length of the magnet, then its dipole moment is given by,

$$\mu_m = m \times l$$

- If an Electric current of *i* amperes flows through a circular wire of one turn having an area of cross section *a* $m^2$, then the magnetic moment is

$$\mu_m = i \times a$$

Unit: ampere (metre)$^2$




Magnetic moment

# Magnetic flux (φ)

- It is defined as the total number of magnetic lines of force passing perpendicular through a given area.

  **Unit: weber**.
- It can also be defined as the total number of lines of force emanating from North Pole.

## Magnetic flux density (or) Magnetic induction (B)

- It is defined as the number of Magnetic Lines of force passing through an unit area of cross section. And it is given by,

$$B = \frac{\text{Magnetic Flux}}{\text{Unit Area}} = \frac{\varphi}{A} \quad \text{weber/m}^2 \text{ (or) Tesla}$$

$$B = \frac{F}{m} = \frac{\text{Force experienced}}{\text{Pole strength}}$$

# Magnetic field strength (or)

# Magnetic field intensity (H)

- Magnetic field intensity or magnetic field strength at any point in a magnetic field is equal to **1 / μ** times the force per pole strength at that point

$$i.e.\ H = \frac{1}{\mu} \times \left( \frac{F}{m} \right) = \frac{B}{\mu}\ ampere\,turns / metre$$

**$\mu$ = permeability of the medium.**

# Magnetization (or)
# Intensity of Magnetization (M)

- Intensity of Magnetization measures the magnetization of the magnetized specimen.

- Intensity of magnetization (*M*) is defined as the Magnetic moment per unit Volume. It is expressed in **ampere/metre.**

# Magnetic susceptibility (χ)

- It is the measure of the ease with which the specimen can be magnetized by the magnetizing force.
- It is defined as the ratio of magnetization produced in a sample to the magnetic field intensity. i.e. magnetization per unit field intensity

$$\chi = \frac{M}{H} \quad (\text{no unit})$$

# Magnetic permeability (μ)

- It is the measure of degree at which the lines of force can penetrate through the material.

- It is defined as the ratio of magnetic flux density in the sample to the applied magnetic field intensity.

$$i.e.\ \mu = \mu_0 \mu_r = \frac{B}{H}$$

- $\mu_0$ = permeability of free space = $4\pi \times 10-7\ H\,m^{-1}$

- $\mu_r$ = relative permeability of the medium

# Relative permeability ($\mu_r$)

- It is the ratio of permeability of the medium to the permeability of free space.

$$\mu_r = \frac{\mu}{\mu_o} \quad \text{(No unit)}$$

## Relation between $\mu_r$ and $\chi$

Total flux density (B) in a solid in the presence of magnetic field can be given as **$B = \mu_0 (H+M)$**

Then $\mu_r$ can be related to $\chi$ as $\mu_r = 1 + \chi$

## Bohr Magneton ($\mu_B$)

- Bohr Magneton is the Magnetic moment produced by one unpaired electron in an atom.

  It is the fundamental quantum of magnetic moment.

$$1 \text{ Bohr Magneton} = \frac{e}{2m} \cdot \frac{h}{2\pi} = \frac{eh}{4\pi m}$$

$$1\mu_B = 9.27 \times 10^{-24} \text{ ampere metre}^2$$

# Current density (J)

- Current density is defined as the ratio of the current to the surface area whose plane is normal to the direction of charge motion.
- The current density is given by,

$$J = \frac{dI}{ds}$$

- The net current flowing through the conductor for the entire surface is

$$I = \int_S J.ds$$

# Conduction Current Density ( $J_1$)

- The current density due to the conduction electrons in a conductor is known as the conduction current density.

- By ohms law, the potential difference across a conductor having resistance R and current I is,

$$V = IR \longrightarrow \quad (1)$$

For a length $l$ and potential difference V,

$$V = El \longrightarrow \quad (2)$$

where E = electric field intensity.

# Expression for $J_1$

From equations V = IR and V= El

$$IR = El \longrightarrow \quad (3)$$

$$R = \rho\frac{l}{A} = \left(\frac{1}{\sigma}\right)\left(\frac{l}{A}\right) \longrightarrow \quad (4)$$

Using (4) in (3)

$$I\left(\frac{l}{\sigma A}\right) = E.l$$

or

$$\frac{I}{\sigma A} = E \quad (\text{or}) \quad \left(\frac{I}{A}\right) = \sigma E \longrightarrow \quad (5)$$

## *Displacement Current Density* $(\overrightarrow{J_2})$

In a capacitor, the current is given by,

$$I_c = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C.\frac{dV}{dt} \longrightarrow (1)$$

In a parallel plate capacitor, the capacitance is given by,

$$C = \frac{\varepsilon A}{d} \longrightarrow (2)$$

Using equation (2) in (1)

$$I_C = \left(\frac{\varepsilon A}{d}\right).\frac{dV}{dt} \ (or) \ \frac{I_C}{A} = \frac{\varepsilon}{d}.\frac{dV}{dt}$$

$J_2$ = Displacement current density = $\varepsilon\left[\frac{d}{dt}\left(\frac{V}{d}\right)\right] = \varepsilon\ \frac{dE}{dt} = \frac{d(\varepsilon E)}{dt}$

$$\overrightarrow{J_2} = \frac{d\overrightarrow{D}}{dt}$$ [since $\overrightarrow{D} = \varepsilon\overrightarrow{E}$ = Electric Displacement vector]

The net current density = **$J = J_1 + J_2$**

$$\overrightarrow{J} = \sigma\overrightarrow{E} + \frac{d\overrightarrow{D}}{dt}$$

# UNIT 3

## EM WAVE CHARACTERISTICS

# Waves in General

**In general, waves are means of transporting energy or information.**

**A wave is a function of both space and time.**

In this chapter, our major goal is to solve Maxwell's equations and derive EM wave motion in the following media:

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_o, \mu = \mu_o$)
2. Lossless dielectrics ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \ll \omega\varepsilon$)
3. Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$)
4. Good conductors ($\sigma \simeq \infty, \varepsilon = \varepsilon_o, \mu = \mu_r\mu_o$, or $\sigma \gg \omega\varepsilon$)

where ω is the angular frequency of the wave. Case 3, for lossy dielectrics, is the most general case and will be considered first. Once this general case is solved, we simply derive other cases (1,2, and 4) from it as special cases by changing the values of *σ, ε, and μ. However, before we consider wave motion in those different media, it is appropriate that* we study the characteristics of waves in general. This is important for proper understanding of EM waves.

Wave motion occurs when a disturbance at point A, at time $t_0$*, is related to what happens at* point *B, at time* $t > t_0$*. A wave equation is a* partial differential equation of the second order. In one dimension, a scalar wave equation takes the form of

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0 \quad \text{----(1)}$$

where *u is the wave velocity.*

in which the medium is source free ($\rho_v$ = 0, J = 0). It can be solved by following procedure,

$E^- = f(z - ut)$ ----(2a) $E^+ = g(z + ut)$ ----(2b) or $E = f(z - ut) + g(z + ut)$ ----(2c)

If we particularly assume harmonic (or sinusoidal) time dependence $e^{j\omega t}$, *eq. (1)* becomes

$$\frac{d^2E_s}{dz^2} + \beta^2 E_s = 0 \quad \text{----(3)}$$

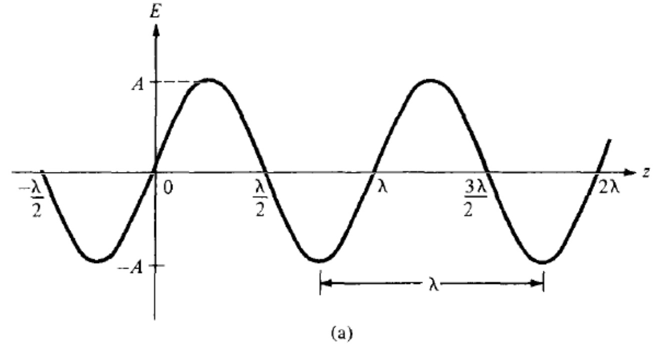
Where β= *ω/u and* $E_s$ *is the phasor form of E. The solutions to eq. (3) are*

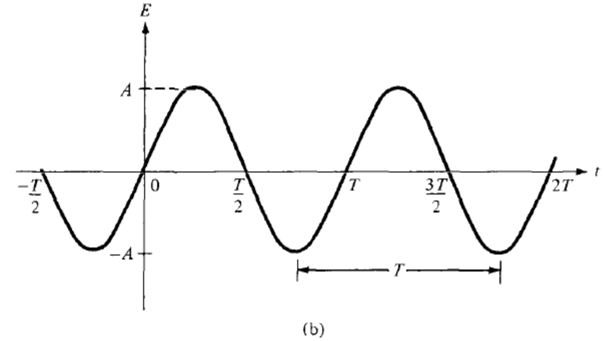
$E^+ = Ae^{j(\omega t - \beta z)}$ ----(4a) $E^- = Be^{j(\omega t + \beta z)}$ ----(4b) and $E = Ae^{j(\omega t - \beta z)} + Be^{j(\omega t + \beta z)}$ ----(4c)

where *A and B are real constants.*
For the moment, let us consider the solution in eq. (4a). Taking the imaginary part of this equation, we have $E = A \sin(\omega t - \beta z)$ ----(5)

This is a sine wave chosen for simplicity; we have taken the real part of eq. (4a).
Note the following characteristics of the wave in eq. (5):
1. It is time harmonic because we assumed time dependence $e^{j\omega t}$ *to arrive at* eq. (5).
2. *A is called the amplitude of the wave and has the same units as E.*
3. *(ωt - βz) is the phase (in radians) of the wave; it depends on time* ***t*** *and space variable* ***z****.*
4. **ω** is the *angular frequency (in radians/second);* ***β*** *is the phase constant or wave number (in radians/meter).*

Due to the variation of *E with both time t and space variable z, we may plot E as a* function of *t by keeping z constant and vice versa.*
*The plots of E(z, t = constant) and E(t, z = constant) are shown in Figure 1(a) and (b), respectively.*
*From Figure 1(a),* we observe that the wave takes distance λ to repeat itself and hence λ is called the *wavelength* (in meters).
From Figure 1(b), the wave takes time **T** *to repeat itself; consequently* **T** *is known as the period (in seconds). Since it takes time* **T** *for the wave to travel* distance λ at the speed *u, we expect* $u = f\lambda$ ----(6a)




**Plot of *E(z, t) =A sin (ωt-βz) (a) with constant t, (b)* with constant *z.***

But *T = l/f, where/is the frequency (the number of cycles per second) of the wave in* Hertz (Hz). Hence, $\lambda = uT$ ----(6b)

Because of this fixed relationship between wavelength and frequency, one can identify the position of a radio station within its band by either the frequency or the wavelength. Usually the frequency is preferred. Also, because

$$\omega = 2\pi f \quad \text{----(7a)}$$

$$\beta = \frac{\omega}{u} \quad \text{----(7b)}$$

and

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \text{----(7c)}$$

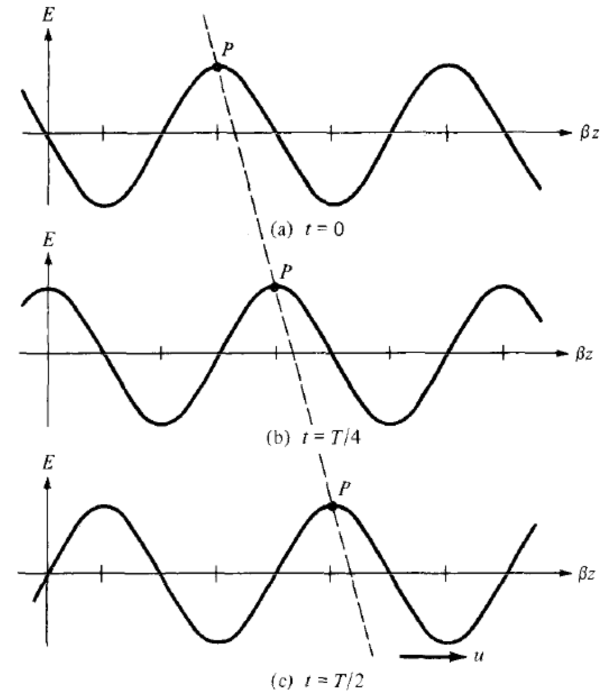
From Eq (6) & (7) We have

$$\beta = \frac{2\pi}{\lambda} \quad \text{----(8)}$$

Equation (8) shows that for every wavelength of distance traveled, a wave undergoes a phase change of $2\pi$ radians.

We will now show that the wave represented by eq. (5) is traveling with a velocity *u in the +z direction. To do this, we consider a fixed point P on the wave. We sketch* eq. (5) at times *t = 0, t/4, and t/2 as in Figure 2. From the figure, it is evident that* as the wave advances with time, point *P moves along +z direction. Point P is a point of* constant phase, therefore

$$\omega t - \beta z = \text{constant} \quad \text{and} \quad \frac{dz}{dt} = \frac{\omega}{\beta} = u \quad \text{----(9)}$$




**Plot of *E(z, t) = A* sin(ωt - βz) at time (a) *t = 0, (b) t = T/4, (c) t = t/2; P moves along +z direction with velocity u.***

In summary, we note the following:

1. A wave is a function of both time and space.
2. Though time t=0 is arbitrarily selected as a reference for the wave, a wave is without beginning or end.
3. A negative sign in *($\omega t \pm \beta z$) is associated with a wave propagating in the +z direction* (forward traveling or positive-going wave) whereas a positive sign indicates that a wave is traveling in the —*z direction (backward traveling or negative going* wave).
4. Since sin *($-\psi$) = $-\sin\psi$ = $\sin(\psi \pm \pi)$, Where as $\cos(-\psi) = \cos\psi$,*

$$\sin(\psi \pm \pi/2) = \pm\cos\psi \quad \text{----(10a)}$$

$$\sin(\psi \pm \pi) = -\sin\psi \quad \text{----(10b)}$$

$$\cos(\psi \pm \pi/2) = \mp\sin\psi \quad \text{----(10c)}$$

$$\cos(\psi \pm \pi) = -\cos\psi \quad \text{----(10d)}$$

Where $\psi = \omega t \pm \beta z$ *With eq. (10), any time-harmonic wave can be represented* in the form of sine or cosine.

# Wave Propagation in Lossy Dielectrics

A lossy **dielectric is a medium in which an EM wave loses power as it propagates** due to poor conduction.

Consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free *($\rho_v$ = 0). Assuming and suppressing the time factor $e^{j\omega t}$, Maxwell's equations becomes*

$$\nabla \cdot \mathbf{E}_s = 0 \quad \text{----(11)} \qquad \nabla \cdot \mathbf{H}_s = 0 \quad \text{----(12)} \qquad \nabla \times \mathbf{E}_s = -j\omega\mu\mathbf{H}_s \quad \text{----(13)}$$

$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s \quad \text{----(14)}$$

Taking the curl of both sides of eq. (13) gives

$$\nabla \times \nabla \times \mathbf{E}_s = -j\omega\mu\, \nabla \times \mathbf{H}_s \quad \text{----(15)}$$

Applying the vector identity $\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$ ----(16)

to the left-hand side of eq. (15) and invoking eqs. (11) and (14), we obtain

$$\nabla(\underbrace{\cancel{\nabla \cdot \mathbf{E}_s}}_{0}) - \nabla^2\mathbf{E}_s = -j\omega\mu(\sigma + j\omega\varepsilon)\mathbf{E}_s \quad \text{or} \quad \nabla^2\mathbf{E}_s - \gamma^2\mathbf{E}_s = 0 \quad \text{----(17)}$$

Where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$ ----(18)

and γ *is called the propagation constant (in per meter) of the medium. By a similar procedure,* it can be shown that for the H field,

$$\nabla^2 \mathbf{H}_s - \gamma^2 \mathbf{H}_s = 0 \quad \text{----(19)}$$

**Equations (17) and (19) are known as homogeneous vector *Helmholtz 's equations or* simply vector *wave equations*.**

*In Cartesian coordinates, eq. (17), for example, is equivalent* to three scalar wave equations, one for each component of E along $a_x$ , *$a_y$ , and $a_z$.* Since γ *in eqs. (17) to (19) is a complex quantity, we may let*

$$\gamma = \alpha + j\beta \quad \text{----(20)}$$

We obtain α *and β from eqs. (18) and (20) by noting that*

$$-\text{Re}\,\gamma^2 = \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \quad \text{----(21)}$$

and $$|\gamma^2| = \beta^2 + \alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} \quad \text{----(22)}$$

From eqs. (21) and (22), we obtain

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]} \quad \text{----(23)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]} \quad \text{----(24)}$$

Without loss of generality, if we assume that the wave propagates along $+a_z$ and that $E_s$ has only an x-component, then

$$\mathbf{E}_s = E_{xs}(z)\mathbf{a}_x \qquad \text{----(25)}$$

Substituting this into eq. (17) yields

$$(\nabla^2 - \gamma^2)E_{xs}(z) \qquad \text{----(26) Hence}$$

$$\underbrace{\frac{\partial^2 E_{xs}(z)}{\partial x^2}}_{0} + \underbrace{\frac{\partial^2 E_{xs}(z)}{\partial y^2}}_{0} + \frac{\partial^2 E_{xs}(z)}{\partial z^2} - \gamma^2 E_{xs}(z) = 0 \qquad \text{or}$$

$$\left[\frac{d^2}{dz^2} - \gamma^2\right]E_{xs}(z) = 0 \qquad \text{----(27)}$$

This is a scalar wave equation, a linear homogeneous differential equation, with solution

$$E_{xs}(z) = E_o e^{-\gamma z} + E'_o e^{\gamma z} \qquad \text{----(28)}$$

where $E_o$ *and* $E'_o$ *are constants. The fact that the field must be finite at infinity requires that* $E'_o = 0$. *Alternatively, because* $e^{\gamma z}$ *denotes a wave traveling along* $-a_z$ *whereas we assume* wave propagation along $a_z$, $E'o = 0$. *Whichever way we look at it,* $E'o = 0$. *Inserting the* time factor $e^{j\omega t}$ *into eq. (28) and using eq. (20), we obtain*

$$\mathbf{E}(z, t) = \text{Re}\,[E_{xs}(z)e^{j\omega t}\mathbf{a}_x] = \text{Re}\,(E_o e^{-\alpha z} e^{j(\omega t - \beta z)}\mathbf{a}_x \quad \boxed{\mathbf{E}(z, t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x} \quad \text{----(29)}$$

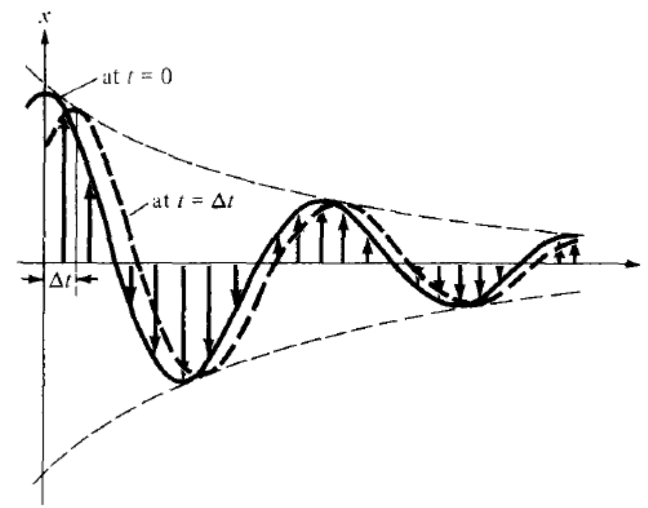
A sketch of |E| at times *t = 0 and t = Δt is portrayed in Figure , where it is evident* that E has only an x-component and it is traveling along the +z direction. Having obtained E(z, *t), we obtain H(z, t) either by taking similar steps to solve eq. (19) or by using eq.* (29) in conjunction with Maxwell's equations. We will eventually arrive at

$$\mathbf{H}(z, t) = \text{Re}\,(H_o e^{-\alpha z} e^{j(\omega t - \beta z)}\,\mathbf{a}_y) \quad \text{----(30)}$$

where $$H_o = \frac{E_o}{\eta} \quad \text{----(31)}$$

And η is a complex quantity known as the ***intrinsic impedance (in ohms)*** *of the medium. It* can be shown by following the steps as

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\underline{/\theta_\eta} = |\eta|e^{j\theta_\eta} \quad \text{----(32)}$$




**E-field with x-component traveling along +z direction at times *t = 0 and t = Δt; arrows indicate instantaneous* values of *E.***

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}, \qquad \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} \quad \text{----(33)}$$

where **0< $\theta_n$<** ***45°.*** *Substituting eqs. (31) and (32) into eq. (30) gives*

$$\mathbf{H} = \mathrm{Re}\left[\frac{E_o}{|\eta| e^{j\theta_\eta}} e^{-\alpha z} e^{j(\omega t - \beta z)}\, \mathbf{a}_y\right] \text{ or } \quad \mathbf{H} = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \mathbf{a}_y \quad \text{----(34)}$$

Notice from eqs. (29) and (34) that as the wave propagates along $a_z$, it decreases or attenuates in amplitude by a factor ***$e^{-\alpha z}$****, and hence a is known as the* ***attenuation constant*** or ***attenuation factor*** *of the medium. It is a measure of the spatial rate of decay of the wave* in the medium, measured in nepers per meter (Np/m) or in decibels per meter (dB/m). An attenuation of 1 neper denotes a reduction to *$e^{-1}$ of the original value whereas an increase* of 1 neper indicates an increase by a factor of *e. Hence, for voltages*

$$1 \text{ Np} = 20 \log_{10} e = 8.686 \text{ dB} \quad \text{----(35)}$$

We also notice from eqs. (29) and (34) that **E** and **H** are out of phase by $\theta_n$, at any instant of time due to the complex intrinsic impedance of the medium. **Thus at any time, E leads H (or H lags E) by $\theta_n$**. *Finally, we notice that the ratio of the magnitude of the conduction* current density J to that of the displacement current density ***$J_d$*** *in a lossy medium* is

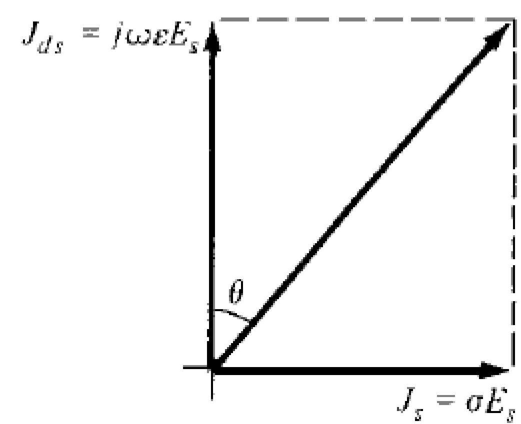
$$\frac{|\mathbf{J}_s|}{|\mathbf{J}_{ds}|} = \frac{|\sigma \mathbf{E}_s|}{|j\omega\varepsilon \mathbf{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta \qquad \text{or} \qquad \tan\theta = \frac{\sigma}{\omega\varepsilon} \qquad \text{----(36)}$$

where tan$\vartheta$ *is known as the loss tangent and* $\vartheta$ *is the loss angle of the medium as illustrated* in Figure. Although a line of demarcation between good conductors and lossy dielectrics is not easy to make, tan$\vartheta$ *or* $\vartheta$ *may be used to determine how lossy a medium is.*

**A medium is said to be a good (lossless or perfect) dielectric if tan $\vartheta$ *is very small* ($\sigma \ll \omega\varepsilon$) *or a good conductor if tan* $\vartheta$ *is very large* ($\sigma \gg \omega\varepsilon$).**

***From the viewpoint of* wave propagation, the characteristic behavior of a medium depends not only on its constitutive parameters σ, *ε and μ but also on the frequency of operation.***

***A medium that is regarded* as a good conductor at low frequencies may be a good dielectric at high frequencies.**



# PLANE WAVES IN LOSSLESS DIELECTRICS

In a lossless dielectric, **σ<<ωε**. *We except that*

$$\sigma \simeq 0, \qquad \varepsilon = \varepsilon_o \varepsilon_r, \qquad \mu = \mu_o \mu_r \quad \text{----(37)}$$

Substituting these into eqs. (23) and (24) gives

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\varepsilon} \qquad \text{----(38a)}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \qquad \lambda = \frac{2\pi}{\beta} \qquad \text{----(38b)}$$

Also

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \angle 0^\circ \qquad \text{----(39)}$$

**Thus E and H are in time phase with each other.**

# PLANE WAVES IN FREE SPACE

This is a special case of what we considered

$$\sigma = 0, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o \qquad \text{----(40)}$$

Thus we simply replace ε by $\varepsilon_o$ and μ *by* $\mu_o$ *in eq. (38) or we substitute eq. (40) directly into eqs. (23) and* (24). Either way, we obtain

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} \qquad \text{----(41a)}$$

$$u = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta} \qquad \text{----(41b)}$$

where *$c = 3 \times 10^8$ m/s, the speed of light in a vacuum. The fact that EM wave travels in* free space at the speed of light is significant. It shows that light is the manifestation of an EM wave. In other words, light is characteristically electromagnetic.

By substituting the constitutive parameters in eq. (40) into eq. (33), *$\vartheta_n = 0$ and $\eta = \eta_0$ where $\eta_0$ is called the intrinsic impedance of free space and is given by*

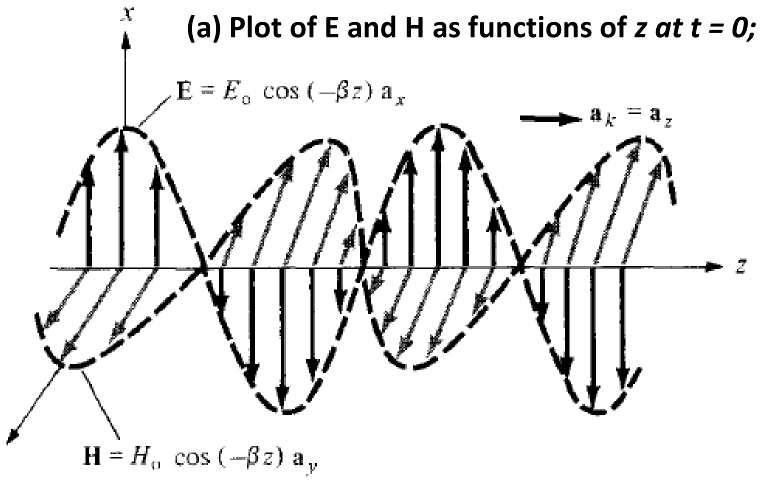
$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \simeq 377\ \Omega \quad \text{----(42)} \qquad \mathbf{E} = E_o\cos(\omega t - \beta z)\,\mathbf{a}_x \ \text{----(43a)} \quad \text{and}$$

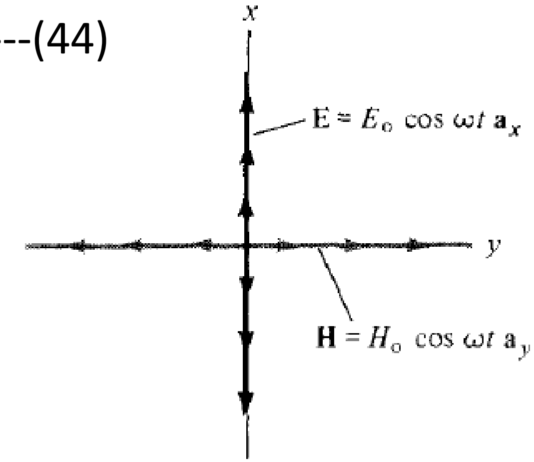
$$\mathbf{H} = H_o\cos(\omega t - \beta z)\,\mathbf{a}_y = \frac{E_o}{\eta_o}\cos(\omega t - \beta z)\,\mathbf{a}_y \quad \text{----(43b)}$$

The plots of E and H are shown in Figure (a). In general, if $a_E$, $a_H$, and $a_k$ *are unit* vectors along the E field, the H field, and the direction of wave propagation;

$$\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H \text{ or } \mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E \text{ or } \boxed{\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k} \quad \text{----(44)}$$

**(a) Plot of E and H as functions of *z at t = 0;***




**(b) plot of E and H at z = 0. The arrows indicate instantaneous values.**

Both E and H fields (or EM waves) are everywhere normal to the direction of wave propagation, $a_k$. *That means, the fields lie in a plane that is transverse or orthogonal to the direction* of wave propagation. They form an EM wave that has no electric or magnetic field components along the direction of propagation; such a wave is called a *transverse electromagnetic* (TEM) wave. Each of E and H is called a *uniform plane wave because E (or H)* has the same magnitude throughout any transverse plane, defined by *z = constant. The direction* in which the electric field points is the *polarization of a TEM wave. The wave in* eq. (29), for example, is polarized in the X direction. This should be observed in Figure (b), where an illustration of uniform plane waves is given. A uniform plane wave cannot exist physically because it stretches to infinity and would represent an infinite energy. However, such waves are characteristically simple but fundamentally important. They serve as approximations to practical waves, such as from a radio antenna, at distances sufficiently far from radiating sources. Although our discussion after eq. (43) deals with free space, it also applies for any other isotropic medium.

# PLANE WAVES IN GOOD CONDUCTORS

This is another special case. A perfect, or good conductor, is one in which **$\sigma >> \omega\varepsilon$** *so that* **$\sigma/\omega\varepsilon \rightarrow \infty$** *that is,* $\sigma \simeq \infty, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o\mu_r$ ----(45)

Hence, eqs. (23) and (24) become

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma} \quad \text{----(46a)}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \qquad \lambda = \frac{2\pi}{\beta} \quad \text{----(46b)}$$

Also $$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \quad \text{----(47)}$$

and thus E leads H by 45°. If

$$\mathbf{E} = E_o e^{-\alpha z}\cos(\omega t - \beta z)\,\mathbf{a}_x \quad \text{----(48a)}$$

then

$$\mathbf{H} = \frac{E_o}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z}\cos(\omega t - \beta z - 45^\circ)\,\mathbf{a}_y \quad \text{----(48b)}$$

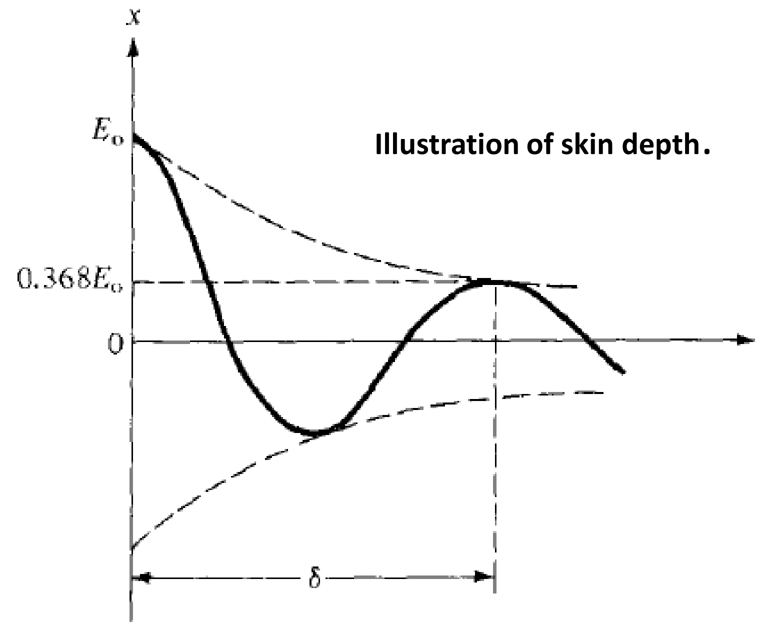



**Illustration of skin depth.**

Therefore, as E (or H) wave travels in a conducting medium, its amplitude is attenuated by the factor ***$e^{-\alpha z}$***.

***The distance δ, through which the wave amplitude* decreases by a factor *$e^{-1}$ (about 37%) is called skin depth or penetration depth of the* medium**; that is

$$E_o e^{-\alpha\delta} = E_o e^{-1}$$

**The skin depth is a measure of the depth to which an EM wave can penetrate the medium.**

or

$$\delta = \frac{1}{\alpha} \qquad \text{----(49a)}$$

For good conductors eqs. (46a) and (49a) give

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \qquad \text{----(49b)}$$

# Poynting Theorem

***This theorem state that the total complex power fed in to volume is equal to the algebraic sum of the active power dissipated as heat plus the reactive power proportional to the difference between time average magnetic & electric energies stored in the volume, plus the complex power transmitted across the surface enclosed by the volume.***

The time average of any two complex vectors is equal to the real part of the product of one complex vector multiplied by the complex conjugate of the other vector
The time average of the instantaneous Poynting vector in steady form is given by

$$\langle P \rangle = \langle \mathbf{E} \times \mathbf{H} \rangle = \tfrac{1}{2}\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \qquad \text{-------(1)}$$

Where $\langle\ \rangle$ stands for average ½ represents complex power when peak values are used, & asterisk ındicates complex conjugate.

The complex Poynting Vector is defined as $\mathbf{P} = \tfrac{1}{2}(\mathbf{E} \times \mathbf{H}^*)$ ------(2)
Maxwell's Equations in Frequency domain are given as follows

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \qquad \text{------(3)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon\mathbf{E} \qquad \text{-----(4)}$$

Dot product of Eq (3) by H* & of conjugate of Eq (4) by E gives

$$(\nabla \times \mathbf{E}) \cdot \mathbf{H}^* = -j\omega\mu \mathbf{H} \cdot \mathbf{H}^* \qquad \text{------(5)}$$

$$(\nabla \times \mathbf{H}^*) \cdot \mathbf{E} = (\mathbf{J}^* - j\omega\epsilon \mathbf{E}^*) \cdot \mathbf{E} \qquad \text{------(6)}$$

Subtracting (5) from (6) results as

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}^*) - \mathbf{H}^* \cdot (\nabla \times \mathbf{E}) = \mathbf{E} \cdot \mathbf{J}^* - j\omega(\epsilon|E|^2 - \mu|H|^2) \quad \text{--(7)}$$

Where E.E* is replaced by $|E|^2$ & H.H* is replaced by $|H|^2$

LHS of Eq (7) is $-\nabla \cdot (\mathbf{E} \times \mathbf{H}^*)$ y vector identity. So we have

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) = -\mathbf{E} \cdot \mathbf{J}^* + j\omega(\epsilon|E|^2 - \mu|H|^2) \quad \text{--(8)}$$

Substituting Eq(2) & (5) in Eq (8) we have

$$-\tfrac{1}{2}\mathbf{E} \cdot \mathbf{J}_0^* = \tfrac{1}{2}\sigma \mathbf{E} \cdot \mathbf{E}^* + j\omega(\tfrac{1}{2}\mu \mathbf{H} \cdot \mathbf{H}^* - \tfrac{1}{2}\epsilon \mathbf{E} \cdot \mathbf{E}^*) + \nabla \cdot \mathbf{P} \qquad \text{-----(9)}$$

Integrating the above equation over the volume and applying Gauss Theorem to the last term on RHS gives

$$\int_v \tfrac{1}{2}(\mathbf{E} \cdot \mathbf{J}_0^*)dv = \int_v \tfrac{1}{2}\sigma|E|^2 dv + j2\omega \int_v (w_m - w_e)dv + \oint_s \mathbf{P} \cdot d\mathbf{s} \quad \text{-(10)}$$

where $\tfrac{1}{2}\sigma|E|^2 = \sigma\langle|E|^2\rangle$ is the time-average dissipated power

$\tfrac{1}{4}\mu \mathbf{H} \cdot \mathbf{H}^* = \tfrac{1}{2}\mu\langle|H|\rangle^2 = w_m$ is the time-average magnetic stored energy

$\tfrac{1}{4}\epsilon \mathbf{E} \cdot \mathbf{E}^* = \tfrac{1}{2}\epsilon\langle|E|^2\rangle = w_e$ is the time-average electric stored energy

$-\tfrac{1}{2}\mathbf{E} \cdot \mathbf{J}_0^* =$ the complex power impressed by the source $\mathbf{J}_0$ into the field

The above equation is known as Complex Poynting Theorem, or Poynting theorem in frequency domain.

Further

$P_{in} = -\int_v \frac{1}{2}(\mathbf{E} \cdot \mathbf{J}_0^*)dv$ be the total complex power suplied by a source within a region

$\langle P_d \rangle = \int_v \frac{1}{2}\sigma|E|^2 dv$ be the time-average power dissipated as heat inside the region

$\langle W_m - W_e \rangle = \int_v (w_m - w_e)dv$ be the difference between time-average magnetic and electric energies stored within the region

$P_{tr} = \oint \mathbf{P} \cdot d\mathbf{s}$ be the complex power transmitted from the region

The above Equation is simplified as follows

$$P_{in} = \langle P_d \rangle + j2\omega[\langle W_m - W_e \rangle] + P_{tr}$$

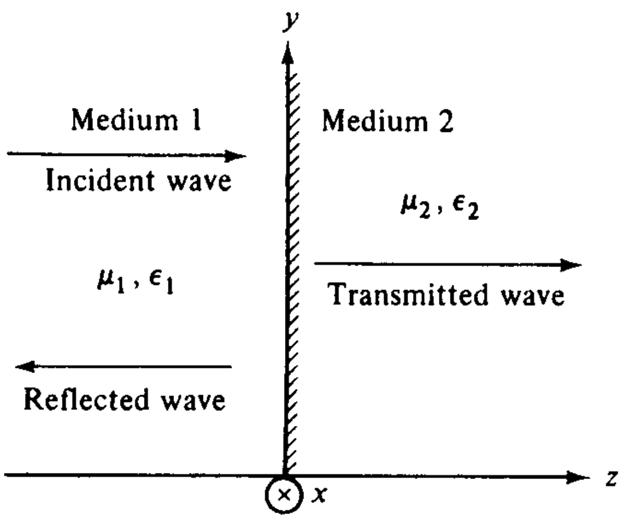
# Uniform Plane Wave Reflection

Reflection of Uniform Plane wave is broadly classified in to two ways

- Normal Incidence
- Oblique Incidence.

## Normal Incidence

The simplest case of reflection is normal incidence it is represented in the following figure. In medium 1 the fields are the sum of incident & reflected waves. So,




----(1)

$$E_x^{(1)} = E_0(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z})$$

$$H_y^{(1)} = \frac{E_0}{\eta_1}(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}) \quad (2)$$

where $\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$

$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{\eta_0}{\sqrt{\epsilon_{r1}}}$ = intrinsic wave impedance of medium 1

$\Gamma$ = reflection coefficient

In medium 2 there are only Transmitted waves

$$E_x^{(2)} = E_0 T e^{-j\beta_2 z} \quad \text{----(3)}$$

$$H_y^{(2)} = \frac{E_0}{\eta_2} T e^{-j\beta_2 z} \quad \text{----(4)}$$

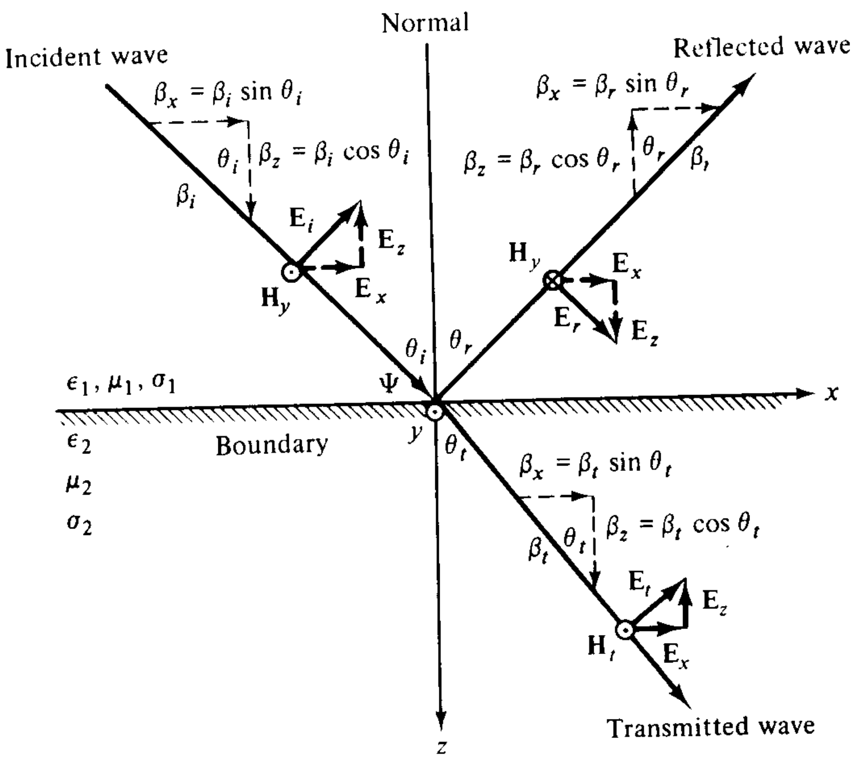
where $\beta_2 = \omega\sqrt{\mu_2\epsilon_2}$

$\eta_2 = \sqrt{\dfrac{\mu_2}{\epsilon_2}} = \dfrac{\eta_0}{\sqrt{\epsilon_{r2}}}$ = intrinsic wave impedance of medium 2

$T$ = transmission coefficient

# Oblique Incidence

**E is in the Plane of Incidence (Parallel polarization)**




When ever a wave is incident obliquely on the boundary surface between media, the polarization of the wave is vertical or horizontal if the electric field is normal or parallel to boundary surface.

Figure shows a loss less dielectric medium. The phase constant of the two media in x direction on the interface are equal as required by the continuity of tangential E & H boundary

We have

$$\beta_i \sin\theta_i = \beta_r \sin\theta_r \quad \text{---(5)}$$

$$\beta_i \sin\theta_i = \beta_t \sin\theta_t \quad \text{---(6)}$$

**H is in the Plane of Incidence (Perpendicular polarization)**

If H is in the plane of incidence the components of H are

$$H_x = H_0 \cos \theta_i e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

$$H_y = 0$$

$$H_z = -H_0 \sin \theta_i e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

The components of Electric intensity E normal to plane of incidence are

$$E_x = 0$$

$$E_y = -\eta_1 H_0 e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

$$E_z = 0$$

The wave impedance in Z direction is given by

$$Z_z = -\frac{E_y}{H_x} = \frac{\eta}{\cos \theta} = \eta \sec \theta$$

# UNIT 4 & 5
# Transmission Lines I & II

# TRANSMISSION LINES

In an electronic system, the delivery of power requires the connection of two wires between the source and the load. At low frequencies, power is considered to be delivered to the load through the wire.

In the microwave frequency region, power is considered to be in electric and magnetic fields that are guided from place to place by some physical structure. Any physical structure that will guide an electromagnetic wave place to place is called a ***Transmission Line***.

Types of Transmission Lines

1. Two wire line
2. Coaxial cable
3. Waveguide
   - Rectangular
   - Circular
4. Planar Transmission Lines
   - Strip line
   - Micro strip line
   - Slot line
   - Fin line
   - Coplanar Waveguide
   - Coplanar slot line

Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). Various kinds of transmission lines such as the twisted-pair and coaxial cables (thinnet and thicknet) are used in computer networks such as the Ethernet and Internet.

**A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be a hydroelectric generator, a transmitter, or an oscillator; the load may be a factory, an antenna, or an oscilloscope, respectively.** Typical transmission lines include coaxial cable, a two-wire line, a parallel-plate or planar line, a wire above the conducting plane, and a microstrip line.

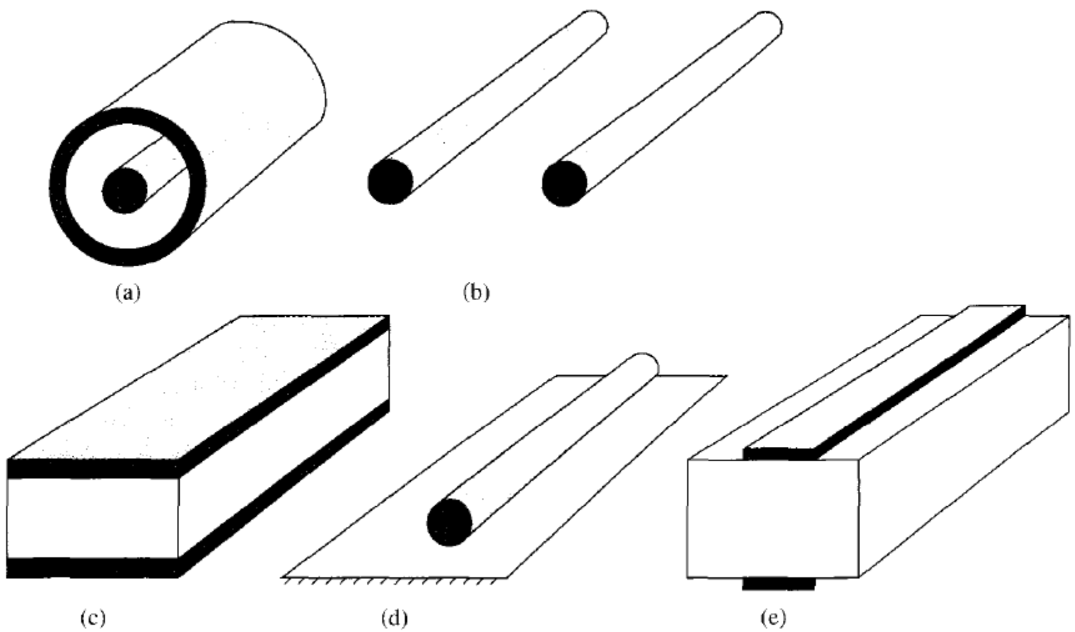



Figure: Cross-sectional view of typical transmission lines: (a) coaxial line, (b) two-wire line, (c) planar line, (d) wire above conducting plane, (e) microstrip line.

At low frequencies, the circuit elements are lumped since voltage and current waves affect the entire circuit at the same time.

At microwave frequencies, such treatment of circuit elements is not possible since voltage and current waves do not affect the entire circuit at the same time.

The circuit must be broken down into unit sections within which the circuit elements are considered to be lumped. This is because the dimensions of the circuit are comparable to the wavelength of the waves according to the formula:

$$\lambda = c/f$$

where,

c = velocity of light

f = frequency of voltage/current

# Electrical Dimensions, Circuit and Field Analysis

## TRANSMISSION LINE PARAMETERS

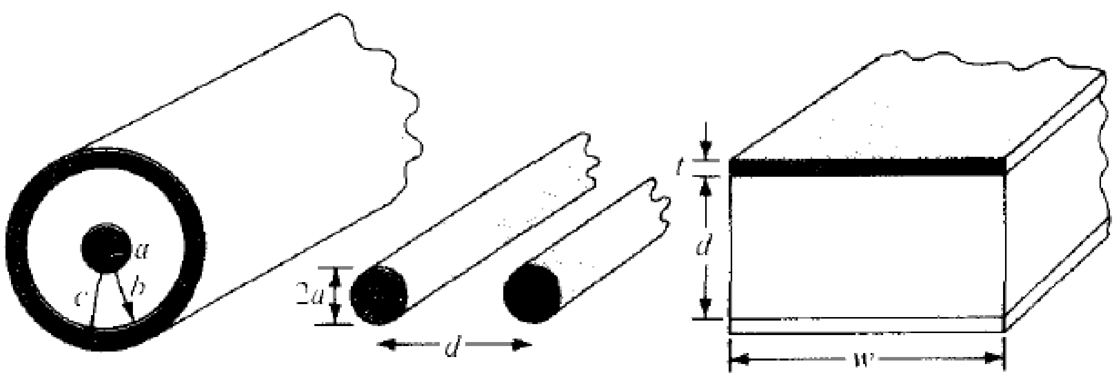
- It is convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length **R**, inductance per unit length **L**, conductance per unit length **G**, and capacitance per unit length **C**.

- Each of the lines has specific formulas for finding R, L, G, and C. For coaxial, two-wire, and planar lines, the formulas for calculating the values of R, L, G, and C are provided in Table. The dimensions of the lines are also shown

# Distributed Line Parameters at High Frequencies

| Parameters | Coaxial Line | Two-Wire Line | Planar Line |
|---|---|---|---|
| $R$ (Ω/m) | $\dfrac{1}{2\pi\delta\sigma_c}\left[\dfrac{1}{a}+\dfrac{1}{b}\right]$ <br> $(\delta \ll a, c-b)$ | $\dfrac{1}{\pi a\delta\sigma_c}$ <br> $(\delta \ll a)$ | $\dfrac{2}{w\delta\sigma_c}$ <br> $(\delta \ll t)$ |
| $L$ (H/m) | $\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$ | $\dfrac{\mu}{\pi}\cosh^{-1}\dfrac{d}{2a}$ | $\dfrac{\mu d}{w}$ |
| $G$ (S/m) | $\dfrac{2\pi\sigma}{\ln\dfrac{b}{a}}$ | $\dfrac{\pi\sigma}{\cosh^{-1}\dfrac{d}{2a}}$ | $\dfrac{\sigma w}{d}$ |
| $C$ (F/m) | $\dfrac{2\pi\varepsilon}{\ln\dfrac{b}{a}}$ | $\dfrac{\pi\varepsilon}{\cosh^{-1}\dfrac{d}{2a}}$ | $\dfrac{\varepsilon w}{d}$ <br> $(w \gg d)$ |

$*\delta = \dfrac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ = skin depth of the conductor; $\cosh^{-1}\dfrac{d}{2a} \simeq \ln\dfrac{d}{a}$ if $\left[\dfrac{d}{2a}\right]^2 \gg 1$.




(a) (b) (c)

Figure: Common transmission lines: (a) coaxial line, (b) two-wire line, (c) planar line.

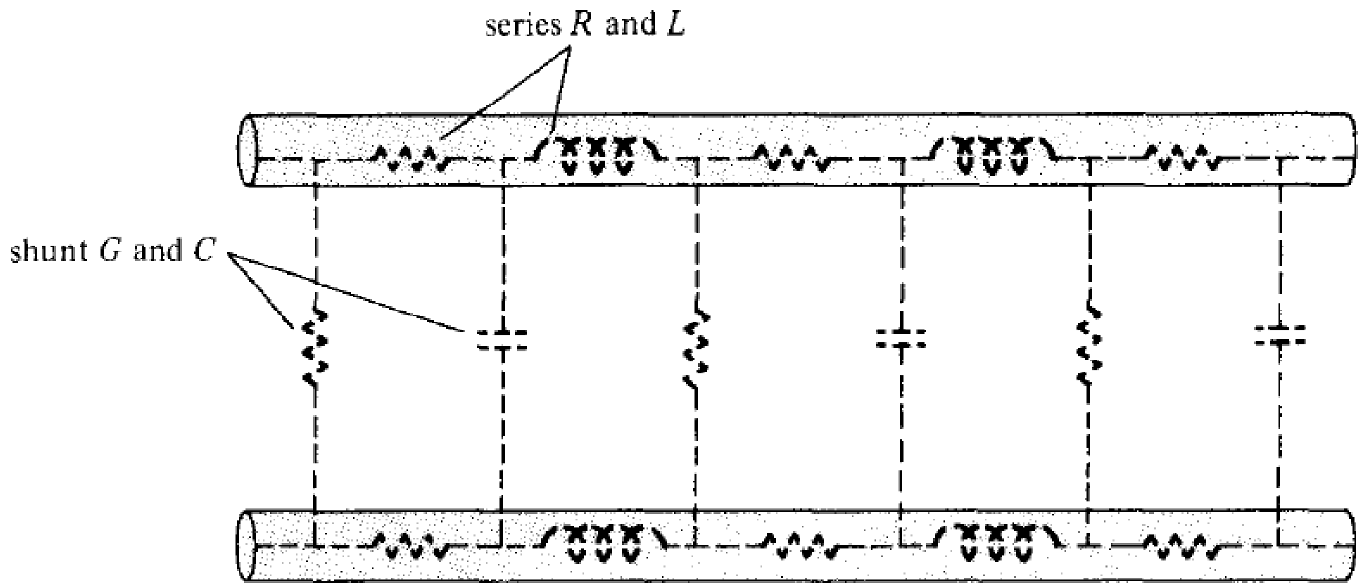
1. The line parameters *R, L, G, and C are not discrete or lumped but distributed as* shown in Figure. By this we mean that the parameters are uniformly distributed along the entire length of the line.

2.For Each Line

$LC = \mu\varepsilon$

and

$$\frac{G}{C} = \frac{\sigma}{\varepsilon}$$

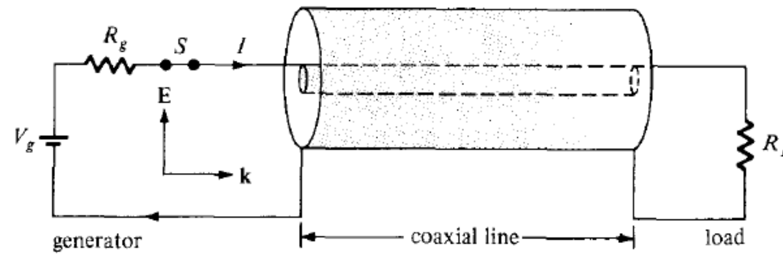



3. For each line, the conductors are characterized by $\sigma_c$ $\mu_c$ $\varepsilon_c = \varepsilon_o$, *and the homogeneous* dielectric separating the conductors is characterized by σ, μ, ε.

4. *$G \neq 1/R$; R is the ac resistance per unit length of the conductors comprising the line* and *G is the conductance per unit length due to the dielectric medium separating* the conductors.

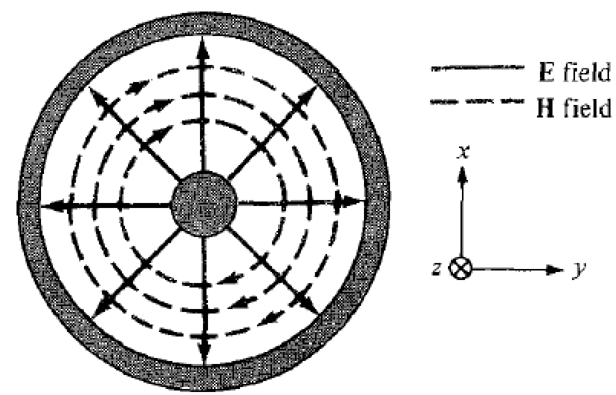
5. The value of *L shown in Table is the external inductance per unit length; that* is, *$L = L_{ext}$. The effects of internal inductance $L_{in}$ (= $R/\omega$) are negligible as high* frequencies at which most communication systems operate.

let us consider how an EM wave propagates through a two-conductor transmission line. For example, consider the coaxial line connecting the generator or source to the load as in Figure (a) . When switch *S is closed,* the inner conductor is made positive with respect to the outer one so that the E field is radially outward as in Figure (b). According to Ampere's law, the H field encircles the current carrying conductor as in Figure (b).



(a) Coaxial line connecting the generator to the load

The Poynting vector (E X H) points along the transmission line. Thus, closing the switch simply establishes a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line. This wave is a non uniform plane wave and by means of it, power is transmitted through the line.

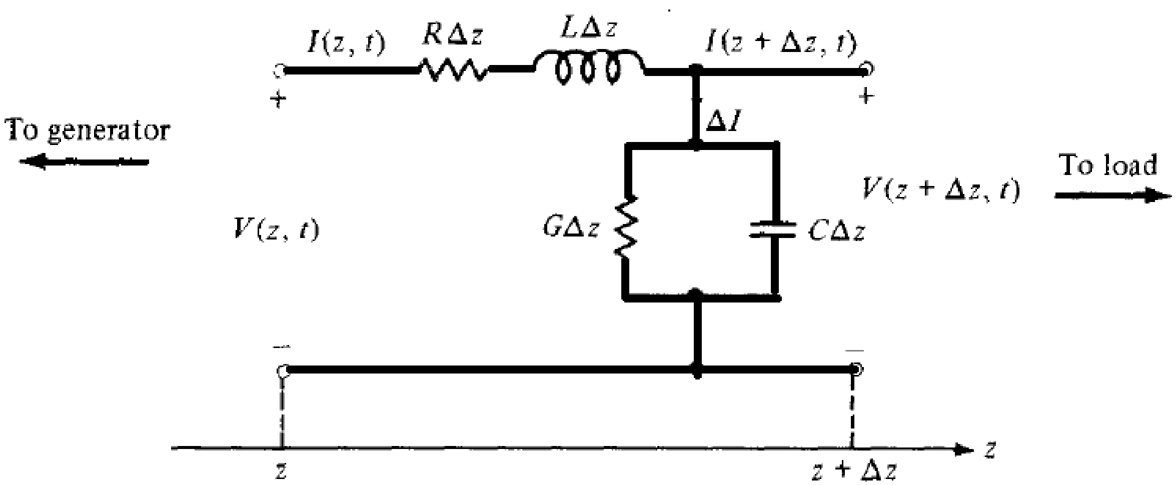


(b) E and H fields on the coaxial line.

# TRANSMISSION LINE EQUATIONS (*lossy type)*

For conductors*($\sigma_c \neq \infty$), For dielectric* ($\sigma \neq 0$).

As mentioned, a two-conductor transmission line supports a TEM wave; that is, the electric and magnetic fields on the line are transverse to the direction of wave propagation. An important property of TEM waves is that the fields E and H are uniquely related to voltage *V and current I, respectively:*




$$V = -\int \mathbf{E} \cdot d\mathbf{l}, \qquad I = \oint \mathbf{H} \cdot d\mathbf{l}$$

------- (1)

Let us examine an incremental portion of length Δ*z of a two-conductor transmission* line. We intend to find an equivalent circuit for this line and derive the line equations.

We expect the equivalent circuit of a portion of the line to be as in Figure above. The model in Figure is in terms of the line parameters *R, L, G, and C,* and may represent any of the two-conductor lines. The model is called the L-type equivalent circuit; there are other possible types. In the model of Figure, we assume that the wave propagates along the +z-direction, from the generator to the load.

By applying Kirchhoff's voltage law to the outer loop of the circuit in Figure, we obtain

$$V(z, t) = R\,\Delta z\, I(z, t) + L\,\Delta z \frac{\partial I(z, t)}{\partial t} + V(z + \Delta z, t)$$

or

$$-\frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = R\, I(z, t) + L \frac{\partial I(z, t)}{\partial t} \quad \text{------- (2)}$$

Taking the limit of eq. (2) as Δz --> 0 leads to

$$-\frac{\partial V(z, t)}{\partial z} = RI(z, t) + L \frac{\partial I(z, t)}{\partial t} \quad \text{------- (3)}$$

Similarly, applying Kirchhoff's current law to the main node of the circuit in Figure gives

$$\begin{aligned} I(z, t) &= I(z + \Delta z, t) + \Delta I \\ &= I(z + \Delta z, t) + G\,\Delta z\, V(z + \Delta z, t) + C\,\Delta z \frac{\partial V(z + \Delta z, t)}{\partial t} \end{aligned}$$

or

$$-\frac{I(z + \Delta z, t) - I(z, t)}{\Delta z} = G\, V(z + \Delta z, t) + C \frac{\partial V(z + \Delta z, t)}{\partial t}$$

$$-\frac{\partial I(z, t)}{\partial z} = G\, V(z, t) + C \frac{\partial V(z, t)}{\partial t} \quad \text{------- (4)}$$

As Δz *—> 0, eq. (4) becomes*

$$-\frac{\partial I(z, t)}{\partial z} = G\, V(z, t) + C \frac{\partial V(z, t)}{\partial t} \quad \text{------- (5)}$$

If we assume harmonic time dependence so that

$$V(z, t) = \text{Re}\,[V_s(z)\, e^{j\omega t}] \quad \text{------- (6a)}$$

$$I(z, t) = \text{Re}\,[I_s(z)\, e^{j\omega t}] \quad \text{------- (6b)}$$

where $V_s(z)$ *and* $I_s(z)$ *are the phasor forms of V(z, t) and I(z, t), respectively, eqs. (3) and* (5) become

$$-\frac{dV_s}{dz} = (R + j\omega L)\, I_s \quad \text{------- (7)}$$

$$-\frac{dI_s}{dz} = (G + j\omega C)\, V_s \quad \text{------- (8)}$$

In the differential eqs. (7) and (8), $V_s$ *and* $I_s$ *are coupled. To separate them, we take* the second derivative of $V_s$ *in eq. (7) and employ eq. (8) so that we obtain*

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)\, V_s \quad \text{or} \quad \frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0 \quad \text{------- (9)}$$

Where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$ ------- (10)

Similarly $\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0$ ------- (11)

We notice that eqs. (9) and (11) are, respectively, the wave equations for voltage and current similar in form to the wave equations obtained for plane waves. Thus, in our usual notations, ϒ *in eq. (10) is the propagation constant* , α *is the attenuation constant (in nepers per meter or decibels[2] per meter), and* β is the phase constant (in radians per meter). The wavelength λ and wave velocity *u are, respectively,* given by

$$\lambda = \frac{2\pi}{\beta} \qquad \text{------- (12)}$$

$$u = \frac{\omega}{\beta} = f\lambda \qquad \text{------- (13)}$$

The solutions of the linear homogeneous differential equations (9) and (11) are

$$V_s(z) = \underset{\longrightarrow\ +z}{V_o^+ e^{-\gamma z}} + \underset{-z\ \longleftarrow}{V_o^- e^{\gamma z}} \qquad \text{------- (14)}$$

$$I_s(z) = \underset{\longrightarrow\ +z}{I_o^+ e^{-\gamma z}} + \underset{-z\ \longleftarrow}{I_o^- e^{\gamma z}} \qquad \text{------- (15)}$$

where *$V_o^+$, $V_o^-$, $I_o^+$, and $I_o^-$ are wave amplitudes; the + and — signs, respectively, denote* wave traveling along *+z and –z directions, as is also indicated by the arrows. Thus, we* obtain the instantaneous expression for voltage as

$$\begin{aligned} V(z, t) &= \text{Re}\,[V_s(z)\, e^{j\omega t}] \\ &= V_o^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_o^- e^{\alpha z} \cos(\omega t + \beta z) \end{aligned} \qquad \text{------- (16)}$$

# Characteristic impedance of a Transmission Line ($Z_o$ )

The characteristic impedance $Z_o$ of the line is the ratio of positively traveling voltage wave to current wave at any point on the line or negatively traveling voltage wave to current wave at any point on the line .

$Z_o$ is analogous to η, the intrinsic impedance of the medium of wave propagation. By substituting eqs. (14) and (15) into eqs. (7) and (8) and equating coefficients of terms *e* $^{\Upsilon z}$ *and e* $^{-\Upsilon z}$*, we obtain*

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} \quad \text{------- (17)}$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_o + jX_o \quad \text{------- (18)}$$

where $R_o$ *and* $X_o$ *are the real and imaginary parts of* $Z_o$*. The propagation constant* ϒ *and the characteristic* impedance $Z_o$ are important properties of the line because they both depend on the line parameters *R, L, G, and C and the frequency of operation. The reciprocal of* $Z_o$ *is the characteristic* admittance $Y_o$*, that is,* $Y_o = 1/Z_O$.

The transmission line considered so far the ***lossy type*** *in that the* conductors comprising the line are imperfect *(*$\sigma_c \neq \infty$*) and the dielectric in which the conductors* are embedded is lossy ($\sigma \neq 0$). Having considered this general case, we may now consider two special cases of lossless transmission line and distortion less line.

# Lossless Line *(R = 0 = G)*

**A transmission line is said lo be lossless if the conductors of the line are perfect** *($\sigma_c = \infty$)* and the dielectric medium separating them is lossless ($\sigma = 0$).

$R = 0 = G$ ------- (19)

This is a necessary condition for a line to be lossless. Thus for such a line, eq. (19) forces eqs. (10), (13), and (18) to become

$$\alpha = 0, \qquad \gamma = j\beta = j\omega\sqrt{LC}$$ ------- (20a)

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$ ------- (20b)

$$X_o = 0, \qquad Z_o = R_o = \sqrt{\frac{L}{C}}$$ ------- (20c)

# Distortion less Line (*R/L = G/C)*

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as *α is frequency dependent.* This results in distortion.

A distortion less line is one in which the attenuation constant *α is frequency independent* while the phase constant β is linearly dependent on frequency. From the general expression for α and β [in eq. (10)], a distortion less line results if the line parameters are such that

***R/L = G/C*** ------- (21)

Thus, for a distortion less line,

$$\gamma = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)} = \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

or $\alpha = \sqrt{RG}, \quad \beta = \omega\sqrt{LC}$ ------- (22a)

Also we have

$$Z_o = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_o + jX_o$$

$$R_o = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \qquad X_o = 0 \qquad \text{------- (22b)}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda \qquad \text{------- (22c)}$$

Note that

1. The phase velocity is independent of frequency because the phase constant β Linearly depends on frequency. We have shape distortion of signals unless *α and u are* independent of frequency.

2. *u and $Z_o$ remain the same as for lossless lines.*

3. A lossless line is also a distortion less line, but a distortion less line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortion less.

# Transmission Line Characteristics

| Case | Propagation Constant $\gamma = \alpha + j\beta$ | Characteristic Impedance $Z_o = R_o + jX_o$ |
|---|---|---|
| General | $\sqrt{(R + j\omega L)(G + j\omega C)}$ | $\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$ |
| Lossless | $0 + j\omega\sqrt{LC}$ | $\sqrt{\dfrac{L}{C}} + j0$ |
| Distortionless | $\sqrt{RG} + j\omega\sqrt{LC}$ | $\sqrt{\dfrac{L}{C}} + j0$ |

**NOTE:**

**For our analysis, we shall restrict our discussion to lossless transmission lines.**

# INPUT IMPEDANCE, SWR, AND POWER

Consider a transmission line of length $l$, characterized by $\gamma$ *and* $Z_o$, *connected to a load* $Z_L$ as shown in Figure. Looking into the line, the generator sees the line with the load as an input impedance $Z_{in}$. It is our intention to determine the input impedance, the standing wave ratio (SWR), and the power flow on the line.

Let the transmission line extend from z = 0 at the generator to z = $l$ at the load. First of all, we need the voltage and current waves in eqs. (14) and (15), that is

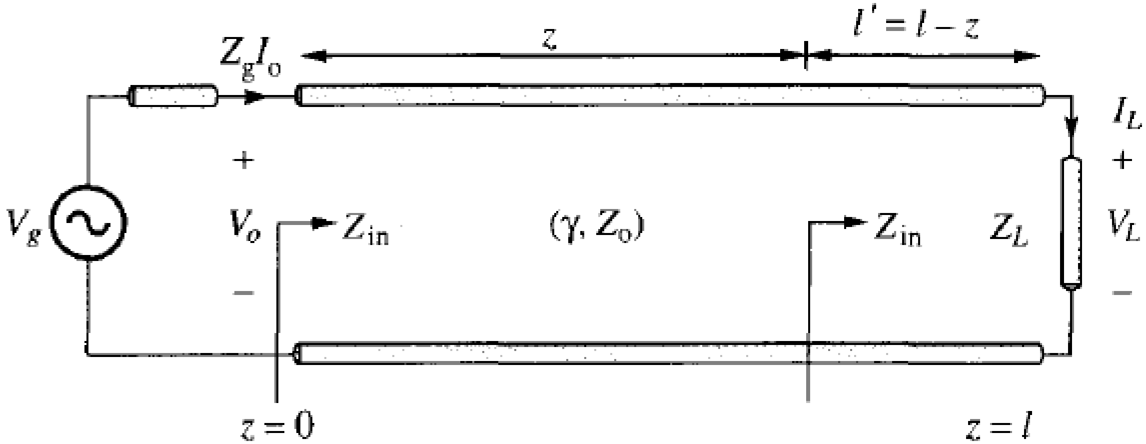
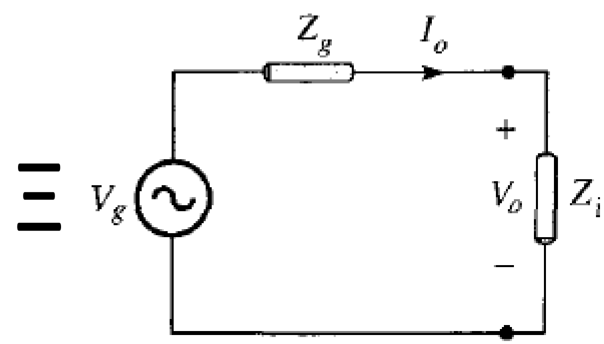



Figure (a) Input impedance due to a line terminated by a load;




(b) equivalent circuit for finding $V_o$ *and* $I_o$ *in terms of* $Z_{in}$ *at* the input.

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \quad \text{------- (23)}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \quad \text{------- (24)}$$

where eq. (17) has been incorporated. To find $\mathbf{V_o^-}$ and $\boldsymbol{V_o^+}$*, the terminal conditions must* be given.

If we are given the conditions at the input, $V_o = V(z = 0), \qquad I_o = I(z = 0)$ ------- (25)

substituting these into eqs. (23) and (24) results in

$$V_o^+ = \frac{1}{2}(V_o + Z_o I_o)$$ ------- (26a)

$$V_o^- = \frac{1}{2}(V_o - Z_o I_o)$$ ------- (26b)

If the input impedance at the input terminals is $Z_{in}$, the input voltage $V_o$ *and the input* current $I_o$ *are easily obtained from Figure (b) as*

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g}$$ ------- (27)

On the other hand, if we are given the conditions at the load, say

$$V_L = V(z = \ell), \qquad I_L = I(z = \ell)$$ ------- (28)

Substituting these into eqs. (23) and (24) gives

$$V_o^+ = \frac{1}{2}(V_L + Z_o I_L)e^{\gamma\ell}$$ ------- (29a)

$$V_o^- = \frac{1}{2}(V_L - Z_o I_L)e^{-\gamma\ell}$$ ------- (29b)

Next, we determine the input impedance $Z_{in}$ = *$V_s(z) / I_s(z)$ at any point on the line. At* the generator, for example, eqs. (23) and (24) yield

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_o(V_o^+ + V_o^-)}{V_o^+ - V_o^-} \quad \text{------- (30)}$$

Substituting eq. (29) into (30) and utilizing the fact that

$$\frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2} = \cosh\gamma\ell, \qquad \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} = \sinh\gamma\ell \quad \text{or} \quad \tanh\gamma\ell = \frac{\sinh\gamma\ell}{\cosh\gamma\ell} = \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{e^{\gamma\ell} + e^{-\gamma\ell}} \quad \text{------- (31)}$$

we get

$$\boxed{Z_{in} = Z_o\left[\frac{Z_L + Z_o\tanh\gamma\ell}{Z_o + Z_L\tanh\gamma\ell}\right]} \quad \text{(lossy)} \quad \text{------- (32)}$$

Although eq. (32) has been derived for the input impedance $Z_{in}$ at the generation end, it is a general expression for finding $Z_{in}$ at any point on the line.

For a lossless line, *$\gamma = j\beta$, $\tanh j\beta l = j\tan\beta l$, and $Z_o = R_o$, so eq. (32) becomes*

$$\boxed{Z_{in} = Z_o\left[\frac{Z_L + jZ_o\tan\beta\ell}{Z_o + jZ_L\tan\beta\ell}\right]} \quad \text{(lossless)} \quad \text{------- (33)}$$

showing that the input impedance varies periodically with distance $l$ from the load. The Quantity $\beta l$ in eq. (33) is usually referred to as the electrical length of the line and can be expressed in degrees or radians.

We now define $\Gamma_L$ *as the voltage reflection coefficient (at the load).* $\Gamma_L$ *is the ratio of* the voltage reflection wave to the incident wave at the load, that is,

$$\Gamma_L = \frac{V_o^- e^{\gamma\ell}}{V_o^+ e^{-\gamma\ell}} \quad \text{------- (34)}$$

Substituting $V_o^-$ *and* $V_o^+$ *m eq. (29) into eq. (34) and incorporating* $V_L = Z_L I_L$ *gives*

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \quad \text{------- (35)}$$

**The voltage reflection coefficient at any point on the line is the ratio of the magnitude of the reflected voltage wave to that of the incident wave.**

$$\Gamma(Z) = \frac{V_o^- e^{\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{2\gamma z}$$

But $z = l - l'$. *Substituting and combining with eq. (34), we get*

$$\Gamma(Z) \frac{V_o^-}{V_o^+} e^{2\gamma l} e^{-2\gamma l'} = \Gamma_L e^{-2\gamma l'} \quad \text{------- (36)}$$

**The current reflection coefficient at any point on the line is negative of the voltage reflection coefficient at that point.**

Thus, the current reflection coefficient at the load is $\frac{I_o^-}{I_o^+} \frac{e^{\gamma l}}{e^{-\gamma l}}$

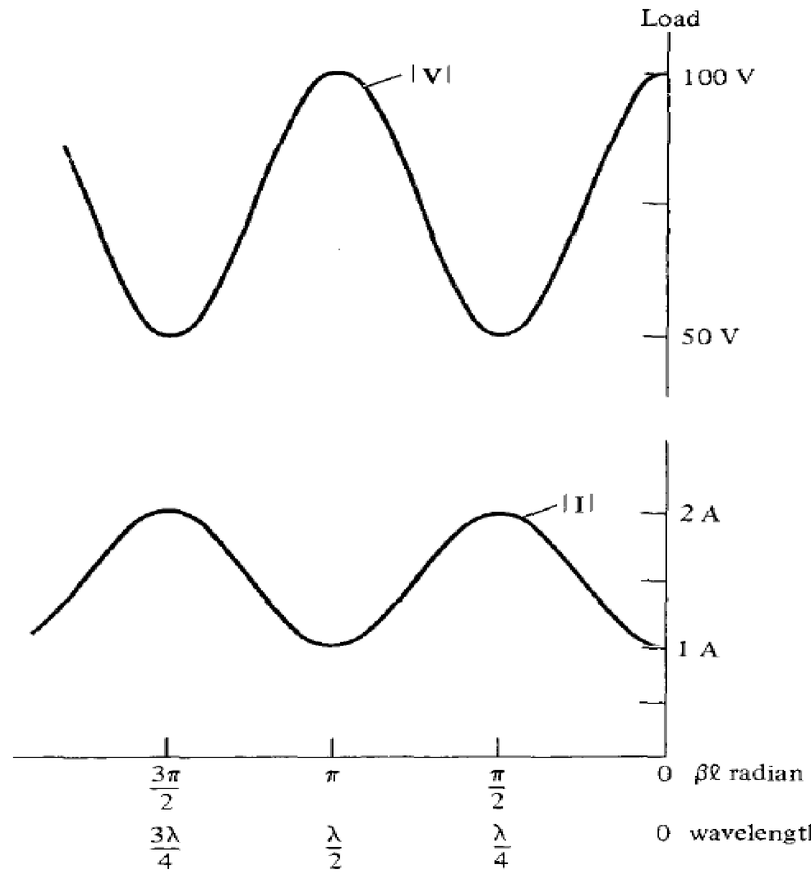
we define the *standing wave ratio s (SWR) as*

$$\frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = S \quad \text{------- (37)}$$

It is easy to show that $I_{max} = V_{max}/Z_o$ and $I_{min} = V_{min}/Z_o$. The input impedance $Z_{in}$ in eq. (33) has maxima and minima that occur, respectively, at the maxima and minima of the voltage and current standing wave. It can also be shown that

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = sZ_o \quad \text{----- (38a)} \qquad \text{and} \qquad |Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_o}{s} \quad \text{----- (38b)}$$

As a way of demonstrating these concepts, consider a lossless line with characteristic impedance of $Z_o$ = 50Ω. *For the sake of simplicity, we assume that the line is terminated* in a pure resistive load *$Z_L$ = 100Ω and the voltage at the load is 100 V (rms). The conditions* on the line are displayed in Figure . Note from the figure that conditions on the line repeat themselves every half wavelength.




Voltage and current wave patterns on a lossless line terminated by a resistive load.

A transmission line is used in transferring power from the source to the load. The average input power at a distance $l$ from the load is given by

$$P_{ave} = \frac{1}{2} Re[V_s(l) I_s^*(l)]$$

where the factor *1/2 is needed since we are dealing with the peak values instead of the rms* values. Assuming a lossless line, we substitute eqs. (23) and (24) to obtain

$$P_{ave} = \frac{1}{2} Re[V_o^+ \left(e^{j\beta l} + \Gamma e^{-j\beta l}\right) \frac{V^{+*}}{Z_o} \left(e^{-j\beta l} - \Gamma^* e^{j\beta l}\right)]$$

$$= \frac{1}{2} Re[\frac{|V_0^+|^2}{z_o} \left(1 - |\Gamma|^2 + \Gamma e^{-2j\beta l} - \Gamma^* e^{2j\beta l}\right)]$$

Since the last two terms are purely imaginary, we have

$$\boxed{P_{ave} = \frac{|V_0^+|^2}{2z_o} (1 - |\Gamma|^2)]} \qquad \text{----- (39)}$$

The first term is the incident power $P_i$*, while the second term is the reflected power* $P_r$. Thus eq. (39) may be written as $P_t = P_i — P_r$

where $P_t$ *is the input or transmitted power and the negative sign is due to the negative going* wave since we take the reference direction as that of the voltage/current traveling toward the right. We should notice from eq. (39) that the power is constant and does not depend on $l$ since it is a lossless line. Also, we should notice that maximum power is delivered to the load when $\Upsilon = 0$*, as expected.*

We now consider special cases when the line is connected to load $Z_L = 0$, $Z_L = \infty$, and $Z_L = Z_o$. *These special cases can easily be derived from the general case.*

## Shorted Line ($Z_L$ = 0)

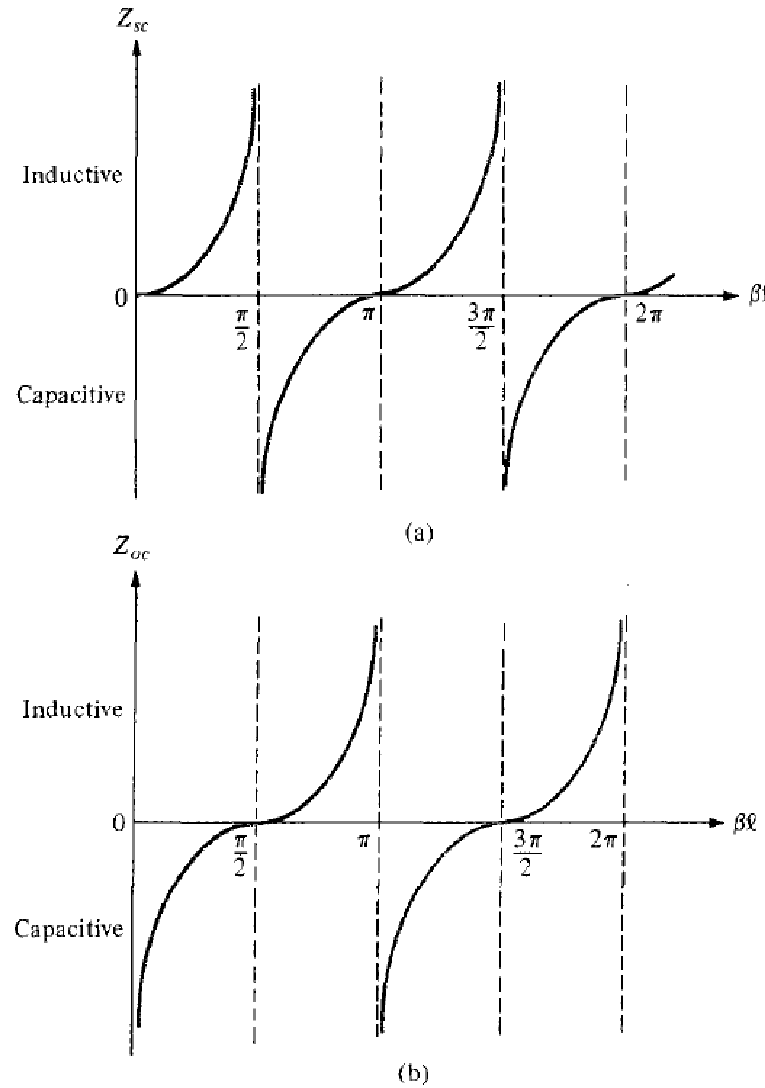
For this case, eq. (33) becomes

$$\boxed{Z_{sc} = Z_{in}\ (at\ Z_L = 0) = jZ_o tan\beta l} \quad \text{----- (40a)}$$

Also

$$\boxed{\Gamma_L = -1, s = \infty} \quad \text{----- (40b)}$$

We notice from eq. (40) that $Z_{in}$ is a pure reactance, which could be capacitive or inductive depending on the value of $l$. *The variation of $Z_{in}$ with $l$ is shown in Figure (a).*



## Open-Circuited Line *($Z_L = \infty$)*

In this case, eq. (33) becomes

$$\boxed{Z_{oc} = \lim_{Z_L \to \infty} (Zin) = \frac{Z_o}{jtan\, \beta l} = -jZ_o \cot \beta l}$$ ----- (41a)

and

$$\boxed{\Gamma_L = 1, \quad s = \infty}$$ ----- (41b)

The variation of $Z_{in}$ with $l$ *is shown in Figure (b). Notice from eqs. (40a) and* (41a) that

$$\boxed{Z_{sc} Z_{oc} = {Z_0}^2}$$ ----- (42)

## Matched Line *($Z_L = Z_o$)*

This is the most desired case from the practical point of view. For this case, eq. (33) reduces to

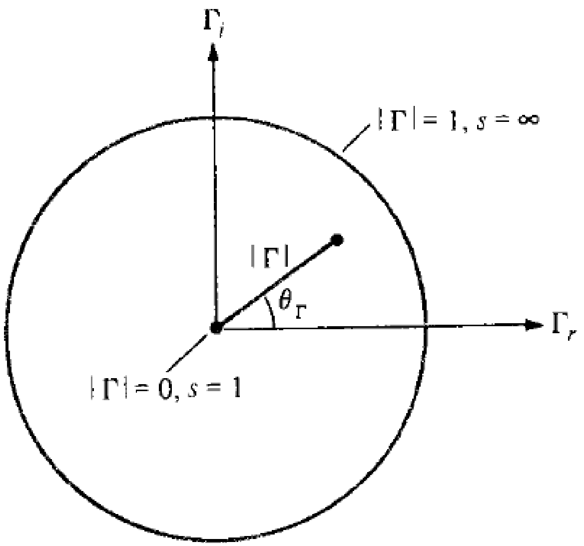
$\boxed{Z_{in} = Z_o}$ ----- (43a) and $\boxed{\Gamma_L = 0, \quad s = 1}$ ----- (43b)

that is, $V_o = 0$, the whole wave is transmitted and there is no reflection. The incident power is fully absorbed by the load. Thus maximum power transfer is possible when a transmission line is matched to the load.

# THE SMITH CHART

Prior to the advent of digital computers and calculators, engineers developed all sorts of aids (tables, charts, graphs, etc.) to facilitate their calculations for design and analysis.

To reduce the tedious manipulations involved in calculating the characteristics of transmission lines, graphical means have been developed. The Smith chart is the most commonly used of the graphical techniques. It is basically a graphical indication of the impedance of a transmission line as one moves along the line. It becomes easy to use after a small amount of experience. We will first examine how the Smith chart is constructed and later employ it in our calculations of transmission line characteristics such as $\Gamma_L$, *s, and* $Z_{in}$. *We* will assume that the transmission line to which the Smith chart will be applied is lossless (Zo = *Ro) although this is not fundamentally required.*




The Smith chart is constructed within a circle of unit radius ($|\Gamma| \le 1$) as shown in Figure. The construction of the chart is based on the relation in eq. 35 that is,

$$\boxed{\Gamma_L} = \boxed{\frac{Z_L - Z_o}{Z_L + Z_o}} \quad \text{----- (44)}$$

or

$$\boxed{\Gamma = |\Gamma| \underline{/\theta_\Gamma} = \Gamma_r + j\Gamma_i} \quad \text{----- (45)}$$

where $\Gamma_r$ and $\Gamma_i$, are the real and imaginary parts of the reflection coefficient $\Gamma$.

Instead of having separate Smith charts for transmission lines with different characteristic impedances such as $Z_o$ = 60,100, and 120 Ω, we prefer to have just one that can be used for any line. We achieve this by using a normalized chart in which all impedances are normalized with respect to the characteristic impedance $Z_o$ of the particular line under consideration. For the load impedance $Z_L$, *for example, the normalized impedance* $Z_L$ *is given* by

$$\boxed{z_L = \frac{Z_L}{Z_o} = r + jx} \quad \text{----- (46)}$$

Substituting eq. (46) into eqs. (44) and (45) gives

$$\boxed{\Gamma = \Gamma_r + j\Gamma_i = \frac{z_L - 1}{z_L + 1}} \quad \text{----- (47a)}$$

or

$$\boxed{z_L = r + jx = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}} \quad \text{----- (47b)}$$

Normalizing and equating components, we obtain

$$\boxed{r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}} \quad \text{----- (48a)}$$

$$\boxed{x = \frac{2\,\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}} \quad \text{----- (48b)}$$

Rearranging terms in eq. (48) leads to

$$\boxed{\left[\Gamma_r - \frac{r}{1 + r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1 + r}\right]^2} \quad \text{----- (49)}$$

and $$[\Gamma_r - 1]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2$$ ----- (50)

Each of eqs. (49) and (50) is similar to $$(x-h)^2 + (y-k)^2 = a^2$$ ----- (51)

which is the general equation of a circle of radius *a, centered at (h, k). Thus eq. (49) is* an *r-circle (resistance circle) with*

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(\frac{r}{1+r}, 0\right)$$ ----- (52a) $$\text{radius} = \frac{1}{1+r}$$ ----- (52b)

For typical values of the normalized resistance r, the corresponding centers and radii of the r-circles are presented in Table. Typical examples of the r-circles based on the data in Table are shown in Figure. Similarly, eq. (50) is an *x-circle (reactance circle)* with

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(1, \frac{1}{x}\right)$$ ----- (53a)

$$\text{radius} = \frac{1}{x}$$ ----- (53b)

| Normalized Resistance ($r$) | Radius $\left(\frac{1}{1+r}\right)$ | Center $\left(\frac{r}{1+r}, 0\right)$ |
|---|---|---|
| 0 | 1 | (0, 0) |
| 1/2 | 2/3 | (1/3, 0) |
| 1 | 1/2 | (1/2, 0) |
| 2 | 1/3 | (2/3, 0) |
| 5 | 1/6 | (5/6, 0) |
| $\infty$ | 0 | (1, 0) |

Typical r-circles for $r = 0, 0.5$, 1,2, 5, $\infty$

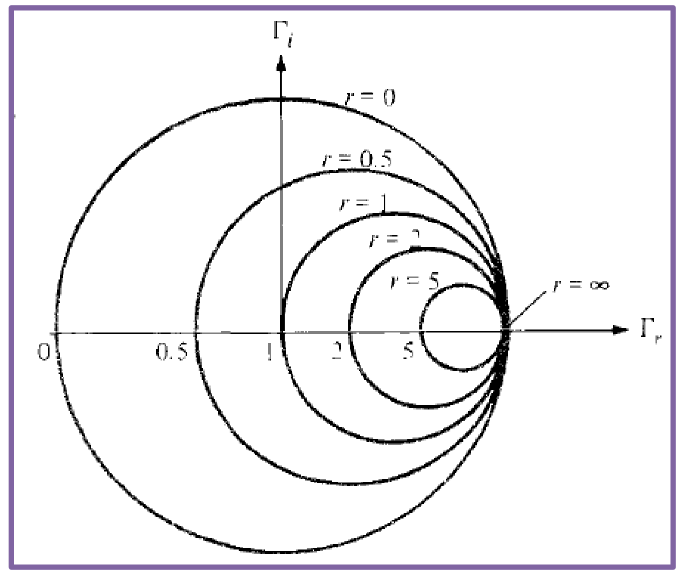
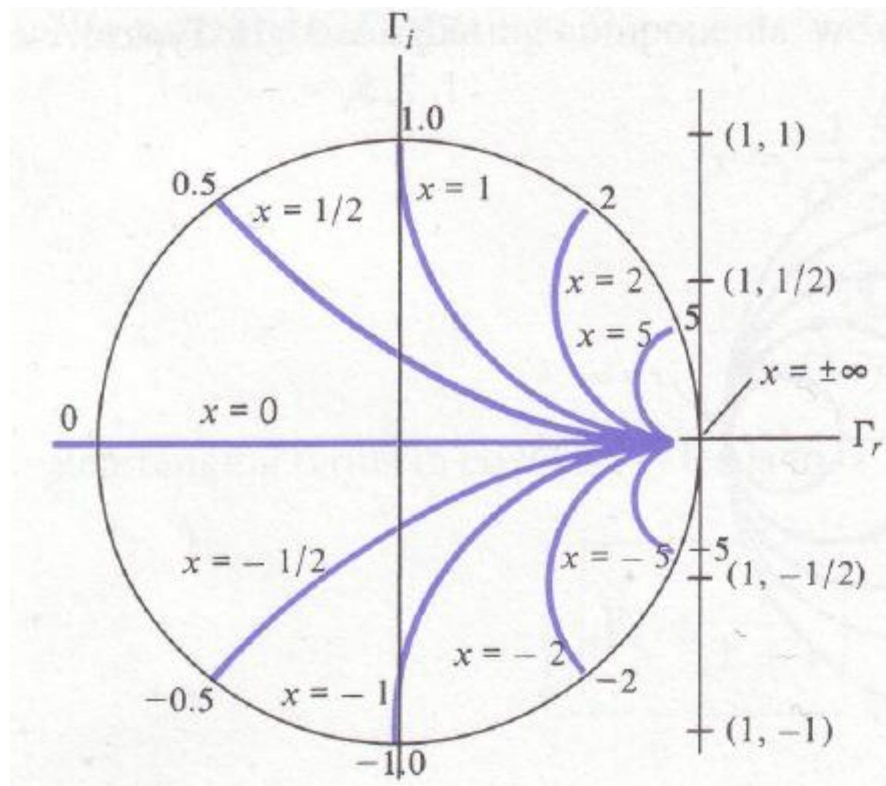


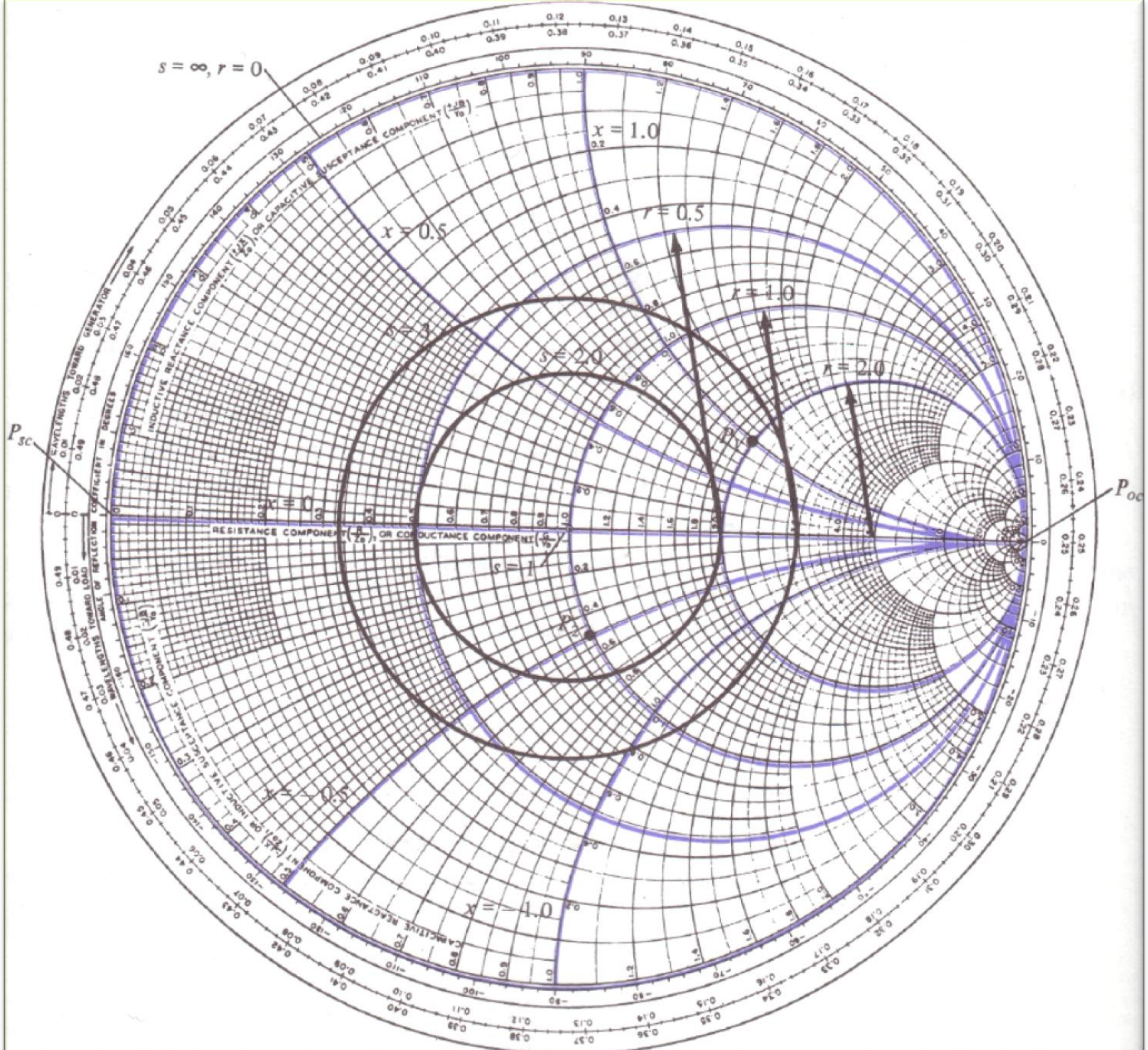
Table below presents centers and radii of the x-circles for typical values of *x, and Figure* below shows the corresponding plots. Notice that while *r is always positive, x can be positive* (for inductive impedance) or negative (for capacitive impedance).

| Normalized Reactance ($x$) | Radius $\left(\frac{1}{x}\right)$ | Center $\left(1, \frac{1}{x}\right)$ |
|---|---|---|
| 0 | $\infty$ | $(1, \infty)$ |
| $\pm 1/2$ | 2 | $(1, \pm 2)$ |
| $\pm 1$ | 1 | $(1, \pm 1)$ |
| $\pm 2$ | 1/2 | $(1, \pm 1/2)$ |
| $\pm 5$ | 1/5 | $(1, \pm 1/5)$ |
| $\pm \infty$ | 0 | $(1, 0)$ |

Typical x circles for *x = 0, ± 1/2,* ±1, ±2, ±5, ±∞.




If we superpose the r-circles and x-circles, what we have is the Smith chart shown in Figure in next slide on the chart, we locate a normalized impedance z = 2 + *j , for example, as* the point of intersection of the *r = 2 circle and the x = 1 circle. This is point $P_1$ in Figure*. Similarly, z = 1 - j0.5 is located at $P_2$*, where the r = 1 circle and the x = -0.5* circle intersect.

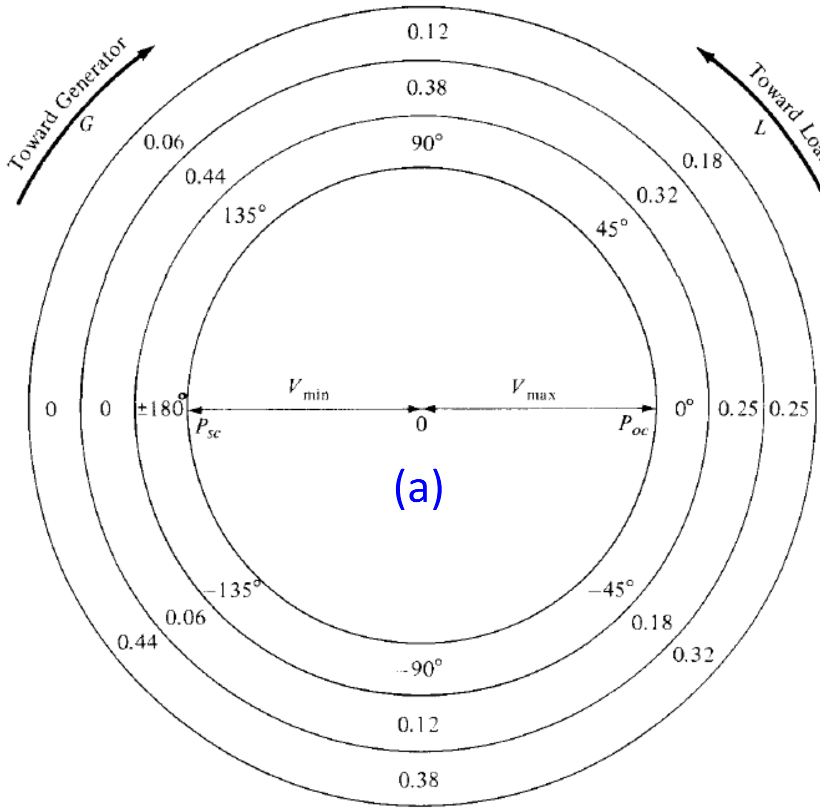

$s = \infty, r = 0$
$x = 1.0$
$x = 0.5$
$r = 0.5$
$r = 1.0$
$r = 2.0$
$s = 3$
$s = 2.0$
$P_{sc}$
$P_{oc}$
$x = 0$
$s = 1$
$x = -0.5$
$x = -1.0$
RESISTANCE COMPONENT $\left(\frac{R}{Z_0}\right)$, OR CONDUCTANCE COMPONENT $\left(\frac{G}{Y_0}\right)$

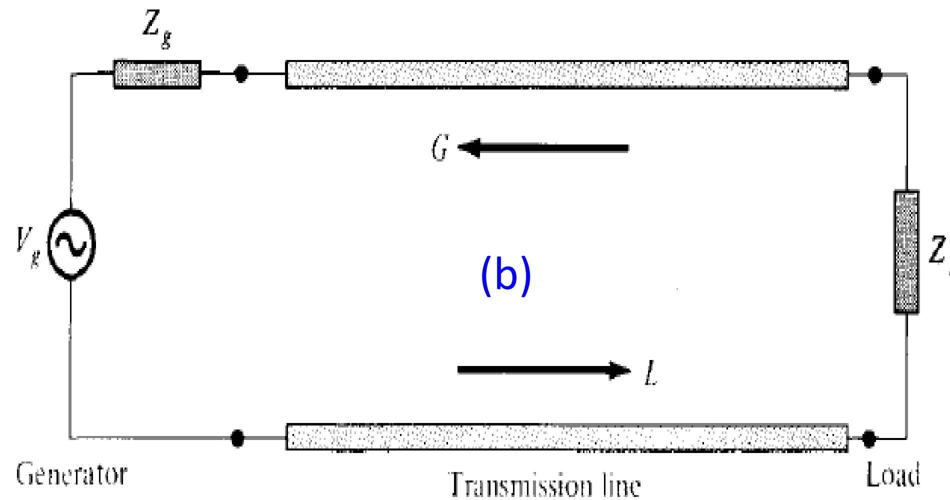
Apart from the r- and x-circles (shown on the Smith chart), we can draw the S-*circles* or *constant standing-wave-ratio circles (always not shown on the Smith chart), which are* centered at the origin with *s varying from 1 to ∞. The value of the standing wave ratio s is* determined by locating where an s-circle crosses the $\Gamma_r$ *axis. Typical examples of S circles* for *S = 1,2, 3, and ∞ are shown in Figure. Since |Γ| and S are related according to* eq. (37), the S circles are sometimes referred to as |Γ|-circles with */Γ/* varying linearly from 0 to 1 as we move away from the center *O toward the periphery of the chart while s* varies nonlinearly from 1 to ∞*.*

The following points should be noted about the Smith chart:

1. At point $P_{sc}$ *on the chart r = 0, x = 0; that is,* $Z_L = 0 + j0$ *showing that* $P_{sc}$ *represents* a short circuit on the transmission line. At point $P_{oc}$, $r = \infty$ *and* $x = \infty$ *or* $Z_L = \infty + j\infty$, which implies that $P_{oc}$ corresponds to an open circuit on the line. Also at $P_{oc}$, *r = 0 and x = 0, showing that* $P_{oc}$ *is another location of a short circuit* on the line.

2. A complete revolution (360°) around the Smith chart represents a distance of λ/2 on the line. Clockwise movement on the chart is regarded as moving toward the generator (or away from the load) as shown by the arrow *G in Figure (a) and* (b). Similarly, counterclockwise movement on the chart corresponds to moving toward the load (or away from the generator) as indicated by the arrow *L in Figure*. Notice from Figure that at the load, moving toward the load does not make sense (because we are already at the load). The same can be said of the case when we are at the generator end.




(a) Smith chart illustrating scales around the periphery and movements around the chart, (b) corresponding movements along the transmission line.

3. There are three scales around the periphery of the Smith chart as illustrated in Figure. The three scales are included for the sake of convenience but they are actually meant to serve the same purpose; one scale should be sufficient. The scales are used in determining the distance from the load or generator in degrees or wavelengths. The outermost scale is used to determine the distance on the line from the generator end in terms of wavelengths, and the next scale determines the distance from the load end in terms of wavelengths. The innermost scale is a protractor (in degrees) and is primarily used in determining $\vartheta_\Gamma$; *it can also be used to determine* the distance from the load or generator. Since a $\lambda/2$ distance on the line corresponds to a movement of 360° on the chart, A *distance on the line corresponds to a 720° movement on the chart.* **$\lambda \rightarrow 720^o$**

Thus we may ignore the other outer scales and use the protractor (the innermost scale) for all our $\vartheta_\Gamma$ *and distance calculations.*

4. $V_{max}$ occurs where $Z_{inmax}$ is located on the chart [see eq. (38a)], and that is on the positive $\Gamma_r$ *axis or on* $OP_{OC}$ *in Figure . $V_{min}$ is located at the same point* where we have $Z_{in}$ min on the chart; that is, on the negative $\Gamma_r$ *axis or on* $OP_{sc}$ *in* Figure . Notice that $V_{max}$ and $V_{min}$ (or$Z_{inmax}$ and $Z_{inmin}$) are λ/4 (or 180°) apart.

5.The Smith chart is used both as impedance chart and admittance chart *(Y = 1/Z).* As admittance chart (normalized impedance *y = YIYO = g + jb), the g- and b circles* correspond to *r- and x-circles, respectively.*

Based on these important properties, the Smith chart may be used to determine, among other things, (a) $\Gamma = |\Gamma| \angle \vartheta_{\Gamma}$ *and s; (b)* $Z_{in}$ *or* $Y_{in}$*; and (c) the locations of* $V_{max}$ *and* $V_{min}$ provided that we are given $Z_o$, $Z_L$, *and the length of the line. Some examples will* clearly show how we can do all these and much more with the aid of the Smith chart, a compass, and a plain straightedge.

## TRANSIENTS ON TRANSMISSION LINES

In circuit analysis, when a pulse generator or battery connected to a transmission line is switched on, it takes some time for the current and voltage on the line to reach steady values. This transitional period is called the *transient. The transient behavior just* after closing the switch (or due to lightning strokes) is usually analyzed in the frequency domain using Laplace transform. For the sake of convenience, we treat the problem in the time domain.

Consider a lossless line of length $l$ and characteristic impedance $Z_o$ as shown in Figure (a). Suppose that the line is driven by a pulse generator of voltage $V_g$ *with internal* impedance $Z_g$ *at* $z = 0$ *and terminated with a purely resistive load* $Z_L$*. At the instant* $t = 0$ *that the switch is closed, the starting current "sees" only* $Z_g$ *and* $Z_o$*, so the initial situation* can be described by the equivalent circuit of Figure (b). From the figure, the starting current at $z = 0$, $t = 0^+$ *is given by*

$$I(0, 0^+) = I_o = \frac{V_g}{Z_g + Z_o} \qquad \text{---(1)}$$

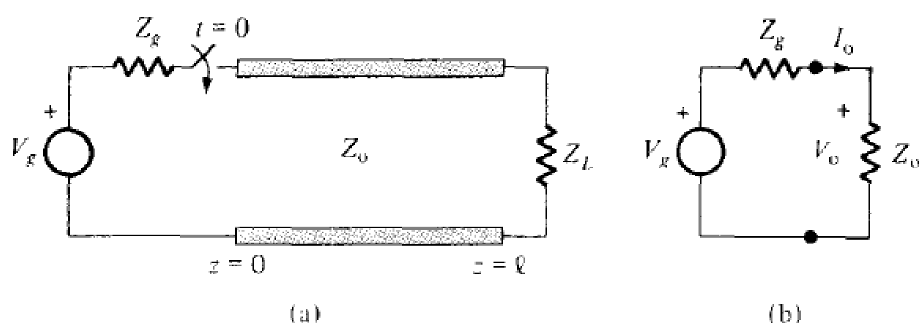



Figure :Transients on a transmission line: (a) a line driven by a pulse generator, (b) the equivalent circuit at *z = 0, t = 0+.*

and the initial voltage is $$V(0, 0^+) = V_o = I_o Z_o = \frac{Z_o}{Z_g + Z_o} V_g \quad \text{---(2)}$$

After the switch is closed, waves $I^+ = I_o$ *and* $V^+ = V_o$ *propagate toward the load at the* speed

$$u = \frac{1}{\sqrt{LC}} \quad \text{---(3)}$$

Since this speed is finite, it takes some time for the positively traveling waves to reach the load and interact with it. The presence of the load has no effect on the waves before the transit time given by

$$t_1 = \frac{\ell}{u} \quad \text{---(4)}$$

After $t_1$ *seconds, the waves reach the load. The voltage (or current) at the load is the sum* of the incident and reflected voltages (or currents). Thus

$$V(\ell, t_1) = V^+ + V^- = V_o + \Gamma_L V_o = (1 + \Gamma_L) V_o \quad \text{---(5)}$$

and

$$I(\ell, t_1) = I^+ + I^- = I_o - \Gamma_L I_o = (1 - \Gamma_L) I_o \quad \text{---(6)}$$

where *$\Gamma_L$ is the load reflection coefficient given that is,* $\boxed{\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}}$

The reflected waves *$V^- = \Gamma_L V_o$ and $I^- = -\Gamma_L I_o$ travel back toward the generator in addition* to the waves *$V_o$ and $I_o$ already on the line. At time $t = 2t_1$ the reflected waves have* reached the generator, so $\boxed{V(0, 2t_1) = V^+ + V^- = \Gamma_G \Gamma_L V_o + (1 + \Gamma_L) V_o}$ ---(7)

or $V(0, 2t_1) = (1 + \Gamma_L + \Gamma_G\Gamma_L)V_o$

and $I(0, 2t_1) = I^+ + I^- = -\Gamma_G(-\Gamma_L I_o) + (1 - \Gamma_L)I_o$

or $I(0, 2t_1) = (1 - \Gamma_L + \Gamma_L\Gamma_G)I_o$

where $\Gamma_G$ is the generator reflection coefficient given by $\boxed{\Gamma_G = \frac{Z_g - Z_o}{Z_g + Z_o}}$

Again the reflected waves (from the generator end) $V^+ = \Gamma_G \Gamma_L V_O$ *and* $I^+ = \Gamma_G \Gamma_L I_O$ *propagate* toward the load and the process continues until the energy of the pulse is actually absorbed by the resistors $Z_g$ *and* $Z_L$.

Instead of tracing the voltage and current waves back and forth, it is easier to keep track of the reflections using a *bounce diagram, otherwise known as a lattice diagram. The* bounce diagram consists of a zigzag line indicating the position of the voltage (or current) wave with respect to the generator end as shown in Figure. On the bounce diagram, the voltage (or current) at any time may be determined by adding those values that appear on the diagram above that time.

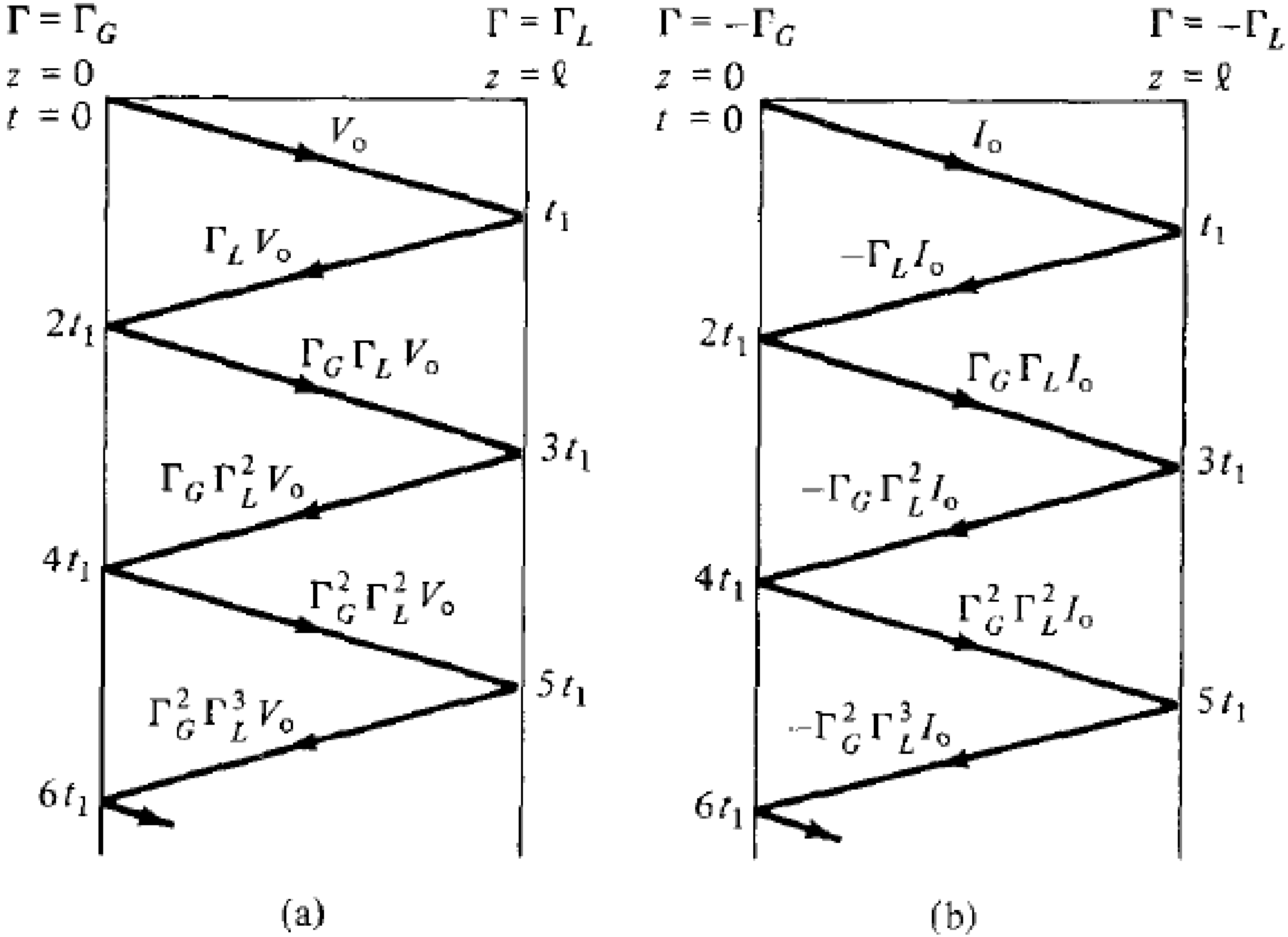



Figure : Bounce diagram for (a) a voltage wave, and (b) a current wave.