KANDULA SRINIVASA REDDY MEMORIAL COLLEGE OF ENGINEERING (AUTONOMOUS)

KADAPA-516003. AP

(Approved by AICTE, Affiliated to JNTUA, Ananthapuramu, Accredited by NAAC)

(An ISO 9001-2008 Certified Institution)

DEPARTMENT OF HUMANITIES & SCIENCES

CERTIFICATE COURSE

ON

"COMPLEX ANALYSIS"

Resource Persons: 1. Dr.G.Radha Associate Professor, Dept.of H&S.
2. Sri.Y.Satheesh Kumar Reddy, Assistant Professor, Dept. of H&S.
3. Sri.G.Sreedhar, Assistant Professor, Dept. of H&S.
4. Sri.B.Veera Sankar,Assistant Professor, Dept. of H&S.
5. Dr.V.Ramachandra Reddy,Assistant Professor, Dept. of H&S.

Course Coordinator: Dr.G.Radha Associate Professor, Dept.of H&S, KSRMCE

Duration : 28/11/2022 to 03/01/2023

K.S.R.M. COLLEGE OF ENGINEERING

(UGC-AUTONOMOUS)

Kadapa,Andhra Pradesh, India– 516 003

Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu.

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Lr. /KSRMCE/ (Humanities & Sciences)/2022-23/

Date: **23.11.2022**

To

The principal,
K.S.R.M.College of Engineering
Kadapa.

From

Dr. G. Radha,
Associate Professor in Mathematics,
Department of H&S,
K.S.R.M College of Engineering,
Kadapa.

Respected Sir,

Sub: KSRMCE - Department of H&S (Mathematics) Permission to conduct Certificate course on Complex Analysis– Request –Reg.

With reference to the above subject, it is brought to your kind notice that the H&S Department is planning to conduct a Certificate Course on **Complex Analysis** for B. Tech III-Sem only CE & ME students from **28th November 2022 to 3rd January 2023** in Offline mode. In this regard I kindly request you sir to grant permission to conduct a certificate course. This is submitted for your kind perusal.

Thanking you Sir,

Yours Faithfully
G. Radha
Dr.G. Radha,
Assoc. professor in Mathematics,
Dept. of H&S,
K.S.R.M.C.E.

Forwarded to principal sir
HoD of H&S.

Permitted
V. S. S. Murthy
23/11/2023

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Cr./KSRMCE/(Department of H&S)/2022-23/ Date:24-11-2022

Circular

It is hereby informed that the Department of H&S is going to conduct a certificate course on Complex Analysis to B. Tech III-Sem (CE &ME) students. This certificate course starts from 28th November 2022 and ends on 3rd January 2023. Interested students may register their names in the staff room Civil block-108 with Dr.G.Radha, Assoc. Prof, Dept. of H&S, (Cell No:9966815484) on or before 26th November 2022.

For any queries contact,

Coordinator

Dr.G.Radha, Assoc. Prof, Dept. of H&S, (Cell No:9966815484)

HoD H&S

Dr. I. SREEVANI M.Sc.,
Professor & HOD
Dept.of Humanities & sciences
K.S.R.M. College of Engineering
KADAPA Dist.

Cc to:

The Management / Dean's/HODs/IQAC / Coordinator for information

Date: 24-11-2022

Name of the Event: Certification Course on Complex Analysis

Venue :CE-205

Registration Form

S.No	Roll Number	Name of the student	Department	Signature
1	219Y1A0115	K. venkata Sujith	Civil	K.V. Sujith
2	219Y1A0137	R. Praveen kumar Reddy	Civil	R. Praveen.
3	229YA0138	P. Sai Kiran	Civil	[illegible]
4	229Y5A0121	G. Venkatesh	Civil	G. [illegible]
5	229Y5A0159	Y. Sivagangadhar Reddy	Civil	[illegible]
6	229Y5A0153	U. Raghavendra	Civil	[illegible]
7	229Y5A0110	C. pavan kumar reddy	civil	[illegible]
8	229Y5A0154	U. Vishnu Vardhan Raju Reddy	Civil	Vishnu
9	219Y1A0126	M. Madhusudhan Reddy	civil	Madhu
10	229Y5A0140	R. Jayanth	Civil	R [illegible]
11	229Y5A0124	K. Sai Kumar reddy	Civil	K. Sai kumar reddy
12	229Y5A0112	C. Madhava	civil	C. Madhava
13	229Y5A0116	D. Farhan Basha	Civil	D. Farhan Basha.
14	229Y5A0136	M. Mohan krishna Naik	civil	M. Mohan
15	229Y5A0157	Y. Prasanna kumar	civil	Y. P kumar

S.No	Roll Number	Name of the student	Department	Signature
16	219Y1A0110	G. Siddartha Naidu	civil	G. Siddartha Naidu.
17	219Y1A0138	R. Lokesh.	Civil	R. Lokesh
18	219Y1A0146	Shaik. Yusuf	civil	Yusuf.
19	219Y1A0145	SMD Hussain	civil	Hus
20	219Y1A0113.	K. Siva Kumar.	civil	[illegible]
21	219Y1A0143	S. mahammad Saif	civil	Saif
22	219Y1A0101	A. L NARASIMHA	Civil	A. L Narasimha
23	219Y1A0161	S. Saikumar.	civil	S. Saikumar
24	219Y1A0162	S. Steevan	civil	S. Steevan
25	219Y1A0118	K. Amarendranath	civil	Amar
26	219Y1A0124	M. chennaiah	civil	M Chenna
27	219Y1A0139	R. chakri	Civil	R chakri
28	219Y1A0157	V. Pranav Reesuit	civil.	[illegible]
29	219Y1A0108	G. uday kiran	civil 3rd sem	G.uday
30	229Y5A0117	D. Bhavana	civil 2nd yr	Bh
31	229Y5A0130	K. Sumithra	civil 2nd yr	Sum
32	229Y5A0144	N. Sashi vardhan	civil 2nd yr	SN
33	229Y5A0133.	M. Sai Teja	civil 2nd yr	M. Sai
34	229Y5A0108	C. Akhila	civil 2nd yr	C. Akhila
35	229Y5A0119	G. GUNAsekhar	civil 2nd year	Guna
36	229Y5A0113	D. Hemanth kumar	civil 2nd year	D. Hem
37	229Y5A0155	U. Venkatesh	civil 2nd year	U. Venkatesh.
38	229Y5A0109	C. Wajid.	civil 2nd year	C. waj
39	229Y5A0123	P. Sharef	civil 2nd year	P. Sharef
40	229Y5A0146	S. Arif	civil	S. Arif

S.No	Roll Number	Name of the student	Department	Signature
41.	229Y5A0104	Bhagya lakshmi	2nd civil	Bhagya
42.	229Y5A0111	C.Ruchitha.	2nd Civil	C.Ruchitha
43	229Y5A0118	E. Abhigna	2nd civil	Abhigna
44.	229Y5A0137	N. Akhila	2nd civil	Akhila
45	229Y5A0105	B.Saisirisha	2nd civil	B.saisirisha
46	229Y5A0102	B. Vamsi Krishna	3rd civil	B. [illegible]
47.	229Y5A0125	K. Varun kumar Reddy	2nd civil	K. Varun kumar

Co-Ordinator

HODH&S

Dr. I. SREEVANI M.Sc., Ph.D
Professor & HOD
Dept.of Humanities & sciences
K.S.R.M. College of Engineering
KADAPA Dist.

Course Title	**COMPLEX ANALYSIS (R20)**	**Certificate Course** CE & ME Branches
Course Objectives: The concepts of complex variables to equip the students to solve application problems.		

Course Outcomes: On successful completion of this course, the students will be able to	
CO 1	**Define** analytic function.
CO 2	**Analyze** images from z-plane to w-plane.
CO 3	**Determine** complex integration along the path.
CO 4	**Define** singularities, poles and residues.
CO 5	**Analyze** real definite integrals by residue theorem.

Module I:

Functions of a complex variable – Limit – Continuity -Differentiability – Analytic function – Properties – Cauchy – Riemann equations in Cartesian and polar coordinates – Harmonic and Conjugate harmonic functions. Construction of analytic function using Milne's - Thomson method.

Module II:

Conformal Mapping: Some standard transforms – translation, rotation, magnification, inversion and reflection. Bilinear transformation – invariant points. Special conformal transformations: w = e^z , z^2, sinz and cosz.

Module III:

Complex integration: Line integral - Evaluation along a path – Cauchy's theorem – Cauchy's integral formula - Generalized integral formula.

Module IV:

Singular point – Isolated singular point – Simple pole, Pole of order *m* – Essential singularity. Residues: Evaluation of residues. Cauchy's residue theorem.

Module V:

Evaluation of the real definite integrals of the type (i) Integration around the unit circle $\int_{0}^{2} f(cos,\ sin)d$ and (ii) integration around a small semi circle $\int_{-\infty}^{\infty} f(x)dx$

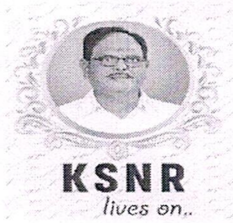
Text books:

1. Higher Engineering Mathematics, Dr. B.S Grewal, Khanna Publishers-42 edition.

K.S.R.M COLLEGE OF ENGINEERING, KADAPA

(Autonomous)

Yerramasupalli, Kadapa, Andhra Pradesh – 516003



Department of Humanities & Sciences

Certification Course

ON

Complex Analysis

Schedule

Date	Timing	Course Instructor	Topic to be covered
28-11-2022	4.00pm-5.00pm	Dr.G. Radha	Introduction to Complex Numbers
29-11-2022	4.00pm-5.00pm	Dr.G. Radha	Functions of a complex variable, Limit, Continuity, Differentiability
30-11-2022	4.00pm-5.00pm	Dr.G. Radha	Analytic function and Properties.
1-12-2022	4.00pm-5.00pm	Dr.G. Radha	Cauchy – Riemann equations in cartesian coordinates
2-12-2022	4.00pm-5.00pm	Dr.G. Radha	Cauchy – Riemann equations in polar coordinates
3-12-2022	2.00pm-5.00pm	Dr.G. Radha	Problem solving based on Cauchy – Riemann equations, Harmonic and Conjugate harmonic functions, Construction of analytic function using Milne's Thomson method, Problem solving based on analytic function.
5-12-2022	4.00pm-5.00pm	Sri. B.Veera Sankar	Some standard transforms – translation, rotation, magnification, inversion and reflection
6-12-2022	4.00pm-5.00pm	Sri. B.Veera Sankar	Bilinear transformation and invariant points
7-12-2022	4.00pm-5.00pm	Sri. B.Veera Sankar	Problem solving based on Bilinear transformation and invariant points
8-12-2022	4.00pm-5.00pm	Sri. B.Veera Sankar	Transformations: $w = e^z, z^2$
9-12-2022	4.00pm-5.00pm	Sri. B.Veera Sankar	Transformations: w = sinz and cosz.
10-12-2022	2:00pm-5.00pm	Dr.V.Ramachandra Reddy	Complex integration: Line integral - Evaluation along a path, Problem solving based on Line integral along a path
12-12-2022	4.00pm-5.00pm	Dr.V.Ramachandra Reddy	Cauchy's theorem
13-12-2022	4.00pm-5.00pm	Dr.V.Ramachandra Reddy	Problem solving based on Cauchy's theorem
14-12-2022	4.00pm-5.00pm	Dr.V.Ramachandra Reddy	Cauchy's integral formula
15-12-2022	4.00pm-5.00pm	Dr.V.Ramachandra Reddy	Generalized integral formula
16-12-2022	4.00pm-5.00pm	Dr.V.Ramachandra Reddy	Singular point, Isolated singular point, Simple pole, Pole of order *m* – Essential singularity

17-12-2022	2.00pm-5.00pm	Sri.Y.Satheesh Kumar Reddy	Evaluation of residues by formula, Problem solving evaluation of residues by formula, Cauchy's residue theorem
19-12-2022	4.00pm-5.00pm	Sri.Y.Satheesh Kumar Reddy	Problem Solving based on Cauchy's residue theorem
20-12-2022	4.00pm-5.00pm	Sri.Y.Satheesh Kumar Reddy	Introduction to evaluation of the real definite integrals of the type
21-12-2022	3.00pm-5.00pm	Sri. G.Sreedhar	Evaluation of the real definite integrals of the type Integration around the unit circle $\int_0^2 f(cos,\ sin)d$
22-12-2022	3.00pm-5.00pm	Sri. G.Sreedhar	Evaluation of the real definite integrals of the type integration around a small semi-circle $\int_{-\infty}^{\infty} f(x)dx$
03-01-2023	3.00pm-5.00pm	Sri. G.Sreedhar	Problem solving based on of the real definite integrals of the type Integration around the unit circle $\int_0^2 f(cos,\ sin)d$ & a small semi-circle $\int_{-\infty}^{\infty} f(x)dx$

Coordinator

HOD H&S

Dr. I. SREEVANI M.Sc., Ph.D
Professor & HOD
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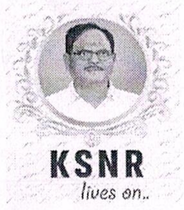

DEPARTMENT OF HUMANITIES AND SCIENCES

Attendance sheet of Certification Course on "Complex Analysis" From 28/11/2022 to 02/01/2023

Sl. No.	Roll No.	Name	28/11	29/ 11	30 /11	1/12	2/12	3/12	5/12	6/12	7/12	8/12	9/12	10/12	12/12	13/12	14/12	15/12	16/12	17/12	19/12	20/12	21/12	22/12	02/01
1.	219Y1A0101	ANAGONDI LAKSHMI NARASIMHA																							
2.	219Y1A0108	GAMPA UDAY KIRAN																							
3.	219Y1A0110	GUNDE SIDDARTHA NAIDU																							
4.	219Y1A0113	KANDLLI SIVA KUMAR																							
5.	219Y1A0115	KOMMALURU VENKATA SUJITH																							
6.	219Y1A0118	KOTHAPALLI AMARENDRANATH																							
7.	219Y1A0124	MOPURI CHENNAIAH																							
8.	219Y1A0126	MURUKUTI MADHUSUDHAN REDDY																							
9.	219Y1A0137	RACHAM REDDY PRAVEEN KUMAR REDDY																							
10.	219Y1A0138	RAMAIAH GARI LOKESH																							

11.	219Y1A0139	RAMANABOINA CHAKRI
12.	219Y1A0142	SEELAM STEEVAN
13.	219Y1A0143	SHAIK KHAN MAHAMMAD SAIF
14.	219Y1A0145	SHAIK MOHAMMED HUSSAIN
15.	219Y1A0146	SHAIK YUSUF
16.	219Y1A0157	VALLAMKONDU PRANAV RASHIK
17.	229Y5A0102	B VAMSI KRISHNA
18.	229Y5A0104	BOGAM BHAGYA LAKSHMI
19.	229Y5A0105	BOYA SAI SIRISHA
20.	229Y5A0108	CHALLA AKHILA
21.	229Y5A0109	CHEEPATI SHAIK WAJIDULLA
22.	229Y5A0110	CHINTHA PAVAN KUMAR REDDY
23.	229Y5A0111	CHUNCHULA RUCHITHA
24.	229Y5A0112	CONDURU MADHAVA
25.	229Y5A0113	DANDUBOINA HEMANTH KUMAR
26.	229Y5A0116	DUDEKULA FARHAN BASHA
27.	229Y5A0117	DUPATI BHAVANA

28.	229Y5A0118	ERUKULA ABHIGNA	[signature]	[signature]	[signature]		[signature]	[signature]		[signature]		[signature]	[signature]	[signature]	[signature]		[signature]	[signature]	[signature]		[signature]	[signature]	[signature]		[signature]
29.	229Y5A0119	GOLLA GUNASEKHAR		[signature]	[signature]	[signature]		[signature]	[signature]	[signature]	[signature]	[signature]		[signature]	[signature]	[signature]		[signature]	[signature]	[signature]	[signature]		[signature]	[signature]	[signature]
30.	229Y5A0121	GUDA VENKATESH	[signature]	[signature]	[signature]	A	[signature]	[signature]	A	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]		[signature]
31.	229Y5A0123	IMMUBAIGARI MAHAMMAD SHARIF	[signature]	[signature]	[signature]	A	[signature]	A	[signature]	[signature]	A	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]
32.	229Y5A0124	KAMBAM SAI KUMAR	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]
33.	229Y5A0130	LANKALA SUMITHRA	[signature]	[signature]	A	[signature]	A	[signature]	A	[signature]	A	[signature]	[signature]	A	[signature]	A	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]
34.	229Y5A0133	MALLEBOYINA SAI TEJA	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]
35.	229Y5A0136	MUDE MOHAN KRISHNA NAIK	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]
36.	229Y5A0137	NAKKALA AKHILA	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]
37.	229Y5A0138	P SAIKIRAN	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]
38.	229Y5A0140	RAJENDRAM JAYANTHACHAR	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]
39.	219Y1A0142	SEELAM STEEVAN	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]
40.	229Y5A0144	SASHI VARDHAN	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	A	[signature]	[signature]	[signature]
41.	229Y5A0146	SHAIK ARIF	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	A	[signature]	[signature]	[signature]
42.	229Y5A0153	ULLASI RAGHAVENDRA	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	A	[signature]	[signature]	[signature]
43.	229Y5A0155	UPPARA VENKATESH	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	A	[signature]	[signature]	[signature]
44.	229Y5A0154	UPENDRAM VISHNU VARDHAN RAJU	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	A	[signature]	[signature]	[signature]	A	A	[signature]	[signature]	[signature]

45.	229Y5A0157	YANABOTHULA PRASANNAKUMAR	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	A	[signature]	[signature]	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]
46.	229Y5A0159	YARRAMREDDY SIVAGANGADHAR REDDY	[signature]	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	A	[signature]	A	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]	[signature]

G. Radha

Coordinator(s)

HoD

Dr. I. SREEVANI M.Sc., Ph.D
Professor & HOD
Dept.of Humanities & sciences
K.S.R.M. College of Engineering
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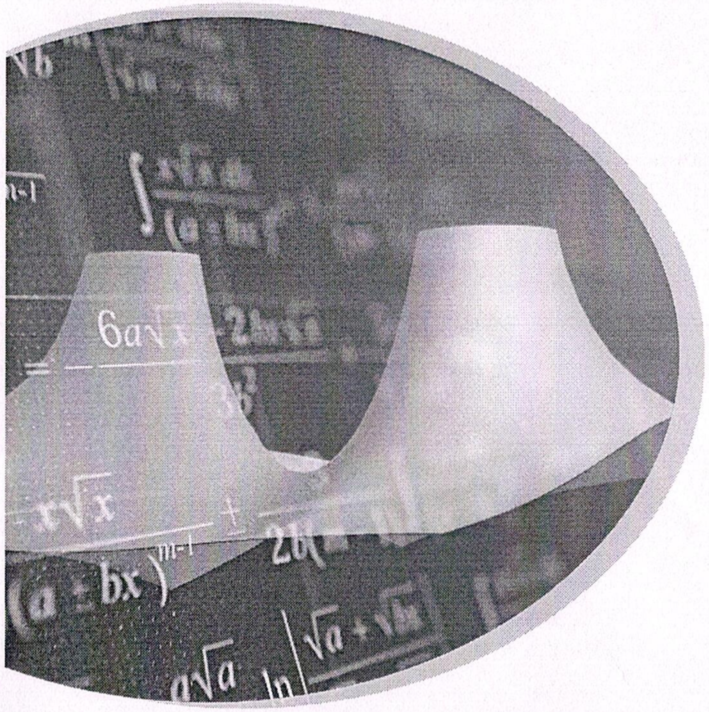
Certification Course on "Complex Analysis"



Department of H & S

CE -205

28th November 2022
to 03rd January 2023

Resource Person **All Mathematics Faculty**

Coordinator **Dr. G. Radha**
Associate Professor

Convenor **Dr. I Sreevani**
HoD Department of H&S

ksrmceofficial www.ksrmce.ac.in 8143731980, 8575697569

K.S.R.M. COLLEGE OF ENGINEERING

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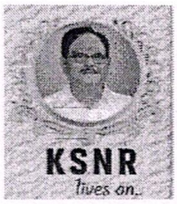


ACTIVITY REPORT

Certification Course

On

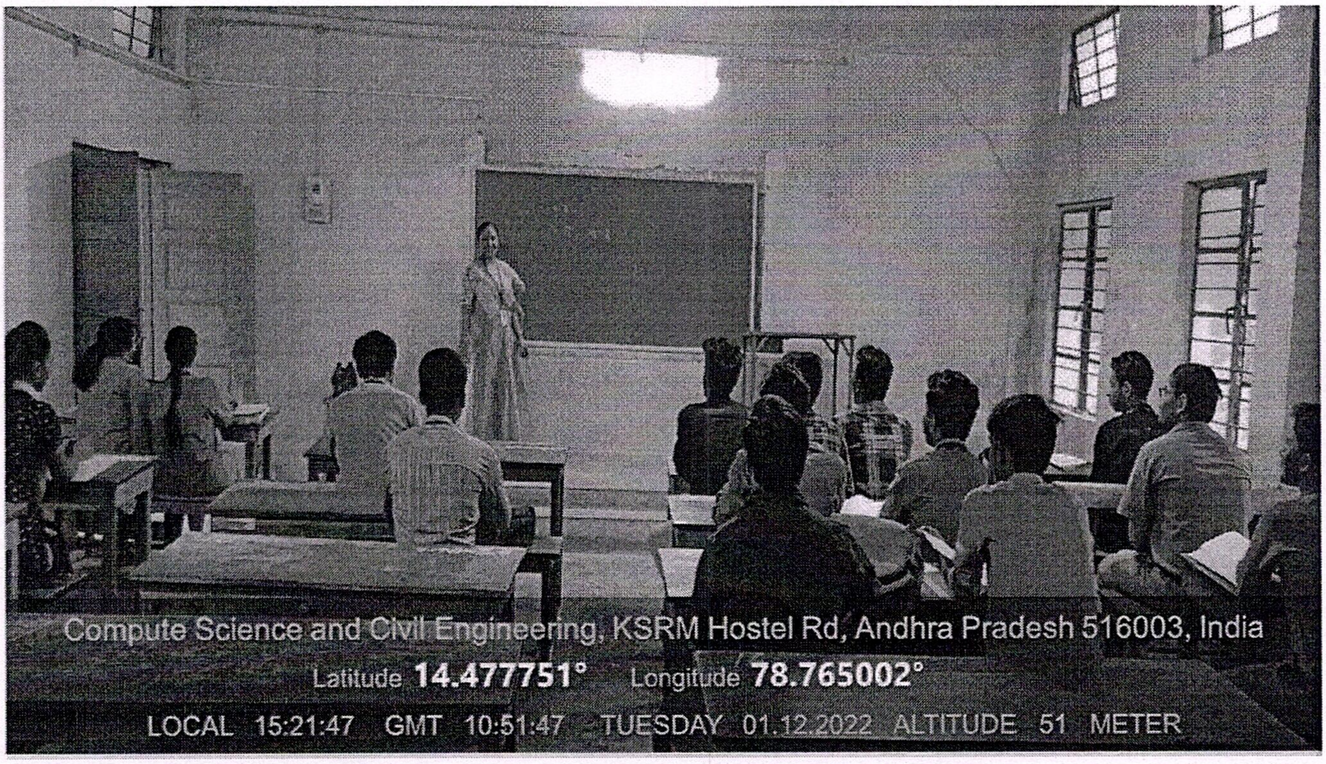
"Complex Analysis"

28th November, 2022 to 03rd January, 2023

Target Group	:	**Students**
Details of Participants	:	**46 Students**
Co-Ordinator	:	**Dr.G. Radha** **Assoc. Prof, Dept. of H&S**
Organizing Department	:	**Department of Humanities & Sciences**
Venue	:	**CE-205**

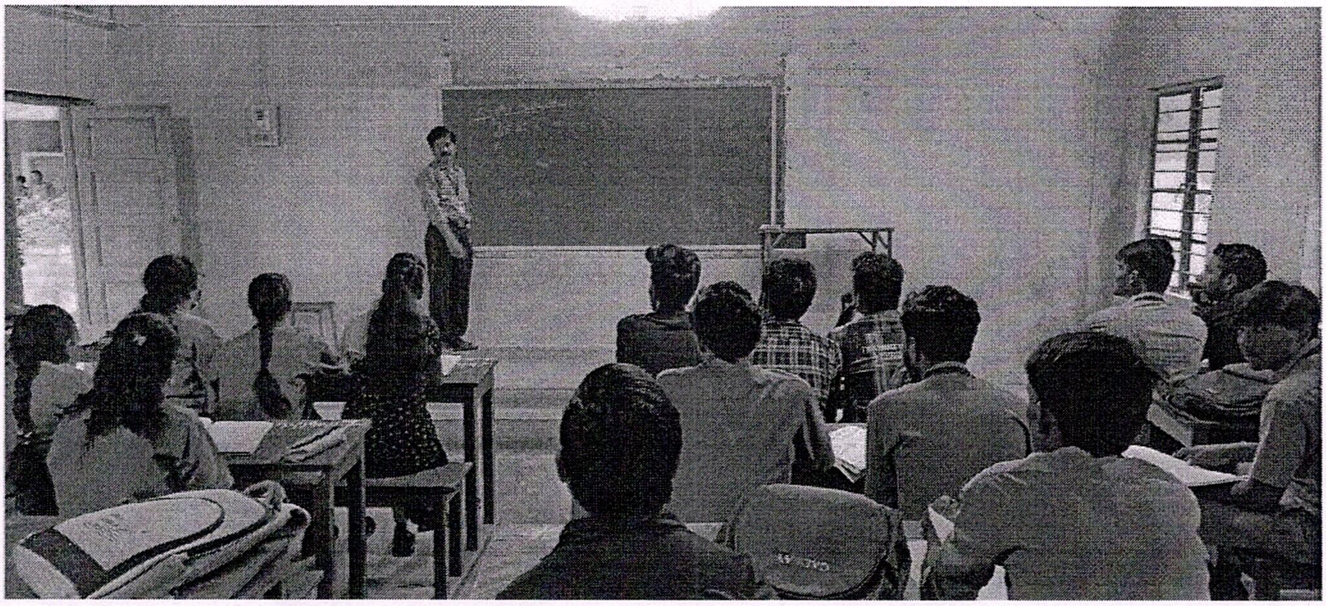
Description: Certification course on Complex Analysis was organized by the Department of Humanities and Sciences from 28th November, 2022 to 03rd January, 2023 offline mode. The entire mathematics faculty acted as Course instructors. The main aim of the course is the study of the techniques of complex variables and functions together with their derivatives, Contour integration and transformations. The course was successfully completed and participation certificates were provided to the participants.

Photos:

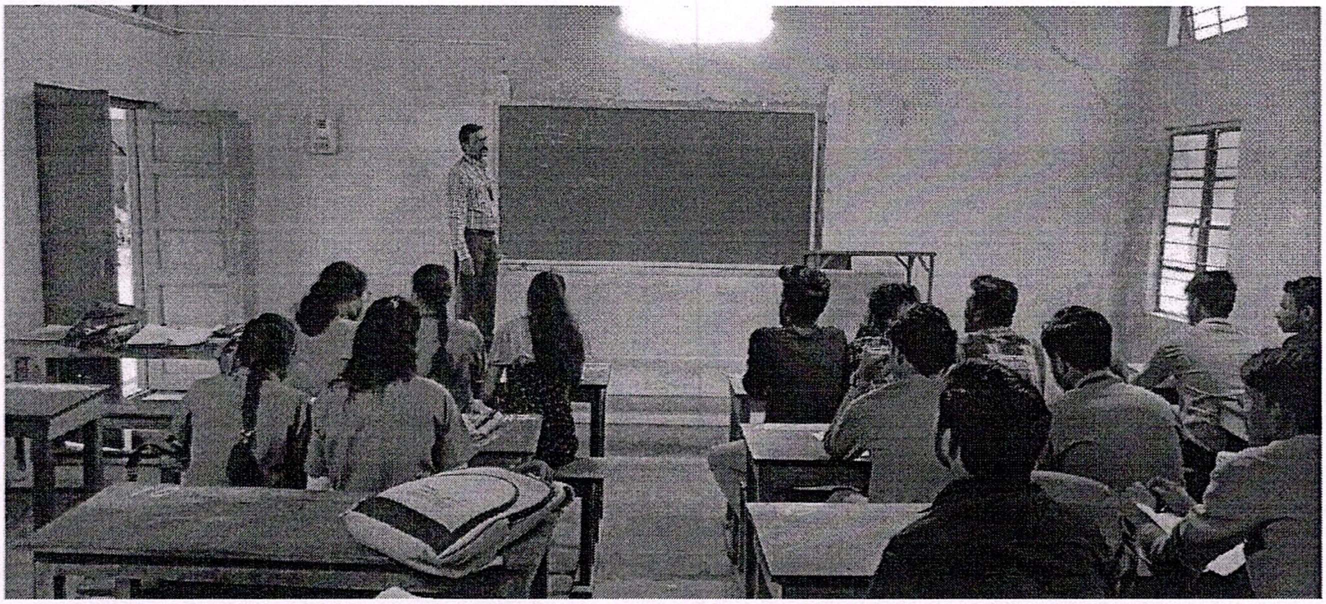


Dr.G.Radha Explained Basic Concepts of Complex Analysis

Sri.B.Veera Sankar Delivered Lecturer on Analytic Function



Sri.G.Sreedhar discussed about the topic Complex Integration



Dr.V.Ramachandra Reddy explained Residues

Coordinator

HoD H&S

Dr. I. SREEVANI M.Sc., Ph.D
Professor & HOD
Dept.of Humanities & sciences
K.S.R.M. College of Engineering
KADAPA Dist.

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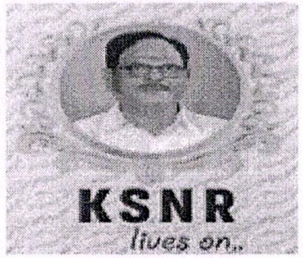

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Certificate of Completion

This is to certify that ---------------------- has successfully completed his certification course on “Complex Analysis” organized by Department of H&S,K.S.R.M.C.E, Kadapa,A.P from ------------

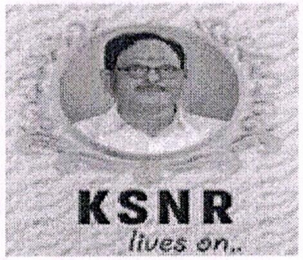
Coordinator

HOD/H&S

Principal

K.S.R.M. COLLEGE OF ENGINEERING

(AUTONOMOUS)



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Certificate of Completion

This is to certify that K.Venkata Sujith (219Y1A0115) has successfully completed his certification course on "Complex Analysis" organized by Department of H&S,K.S.R.M.C.E, Kadapa,A.P from 28/11/2022 to 03/01/2023.

Dr. G.Radha Coordinator	Dr.I.Sreevani HOD/H&S	Dr. V.S.S.Murthy Principal

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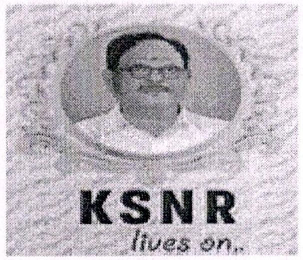

Certificate of Completion

This is to certify that G.Siddartha Naidu (219Y1A0110) has successfully completed his certification course on "Complex Analysis" organized by Department of H&S,K.S.R.M.C.E, Kadapa,A.P from 28/11/2022 to 03/01/2023.

Dr. G.Radha
Coordinator

Dr.I.Sreevani
HOD/H&S

Dr. V.S.S.Murthy
Principal

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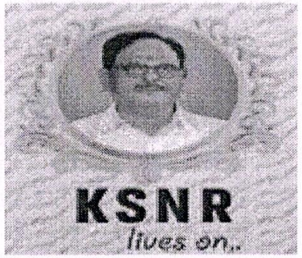

LIGHTED TO LIGHTEN

Certificate of Completion

This is to certify that Y.Siva Gangadhar Reddy (229Y5A0159) has successfully completed his certification course on “Complex Analysis” organized by Department of H&S,K.S.R.M.C.E, Kadapa,A.P from 28/11/2022 to 03/01/2023.

Dr. G.Radha Coordinator	Dr.I.Sreevani HOD/H&S	Dr. V.S.S.Murthy Principal

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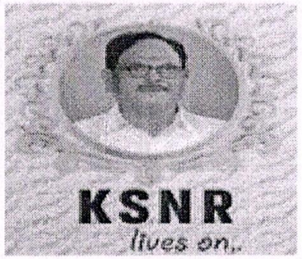

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Certificate of Completion

This is to certify that R.PRAVEEN KUMAR REDDY(219Y1A0137) has successfully completed his certification course on “Complex Analysis” organized by Department of H&S,K.S.R.M.C.E, Kadapa,A.P from 28/11/2022 to 03/01/2023.

Dr. G.Radha Coordinator	Dr.I.Sreevani HOD/H&S	Dr. V.S.S.Murthy Principal

Date: 03-01-2023

Feedback Form on Complex Analysis

Organized by

Department of Humanities & Sciences

1. Is the course contents met your expectations

   [ ] Strongly disagree [ ] Disagree [ ] Agree [ ] strongly agree

2. Rate the content of the course

   [ ] Poor [ ] Satisfactory [ ] Good [ ] Excellent

3. The instructor follow sequence of the content

   [ ] Poor [ ] Satisfactory [ ] Good [ ] Excellent

4. Is the speaker illustrated topics with adequate examples

   [ ] yes [ ] No

5. The instructors encouraged interaction and were helpful

   [ ] Poor [ ] Satisfactory [ ] Good [ ] Excellent

Signature of the participant

Date: 03-01-2023

Feedback Form on Complex Analysis

Organized by

Department of Humanities & Sciences

1. Is the course contents met your expectations
   [ ] Strongly disagree [ ] Disagree [ ] Agree [✓] strongly agree
2. Rate the content of the course
   [ ] Poor [ ] Satisfactory [ ] Good [✓] Excellent
3. The instructor follow sequence of the content
   [ ] Poor [ ] Satisfactory [✓] Good [ ] Excellent
4. Is the speaker illustrated topics with adequate examples
   [✓] yes [ ] No
5. The instructors encouraged interaction and were helpful
   [ ] Poor [ ] Satisfactory [ ] Good [✓] Excellent

K. Venkata Sujith

Signature of the participant

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KSNR lives on...

Date: 03-01-2023

Feedback Form on Complex Analysis

Organized by

Department of Humanities & Sciences

1. Is the course contents met your expectations

   [ ] Strongly disagree [ ] Disagree [ ] Agree [✓] strongly agree

2. Rate the content of the course

   [ ] Poor [ ] Satisfactory [✓] Good [ ] Excellent

3. The instructor follow sequence of the content

   [ ] Poor [ ] Satisfactory [ ] Good [✓] Excellent

4. Is the speaker illustrated topics with adequate examples

   [✓] yes [ ] No

5. The instructors encouraged interaction and were helpful

   [ ] Poor [ ] Satisfactory [✓] Good [ ] Excellent

N. Akhila

Signature of the participant

K.S.R.M. COLLEGE OF ENGINEERING

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KSNR lives on..

Date: 03-01-2023

Feedback Form on Complex Analysis

Organized by

Department of Humanities & Sciences

1. Is the course contents met your expectations

   [ ] Strongly disagree [ ] Disagree [ ] Agree [✓] strongly agree

2. Rate the content of the course

   [ ] Poor [ ] Satisfactory [ ] Good [✓] Excellent

3. The instructor follow sequence of the content

   [ ] Poor [ ] Satisfactory [✓] Good [ ] Excellent

4. Is the speaker illustrated topics with adequate examples

   [✓] yes [ ] No

5. The instructors encouraged interaction and were helpful

   [ ] Poor [ ] Satisfactory [ ] Good [✓] Excellent

L. sumithra

Signature of the participant

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KSNR lives on..

DEPARTMENT OF HUMANITIES AND SCIENCES

Feedback Analysis of Certification Course on "Complex Analysis" From 28/11/2022 to 02/01/2023

S. No.	Roll No.	Name of the Student	1. Is the course contents met your expectations	2.Rate the content of the course	3. The instructor follow sequence of the content	4.Is the speaker illustrated topics with adequate examples	5.The instructors encouraged interaction and were helpful
1.	219Y1A0101	ANAGONDI LAKSHMI NARASIMHA	strongly agree	Good	Good	Yes	Excellent
2.	219Y1A0108	GAMPA UDAY KIRAN	Excellent	Good	Good	Yes	Excellent
3.	219Y1A0110	GUNDE SIDDARTHA NAIDU	strongly agree	Good	Excellent	Yes	Good
4.	219Y1A0113	KANDLLI SIVA KUMAR	strongly agree	Good	Good	Yes	Excellent
5.	219Y1A0115	KOMMALURU VENKATA SUJITH	strongly agree	Excellent	Good	Yes	Excellent
6.	219Y1A0118	KOTHAPALLI AMARENDRANATH	strongly agree	Good	Good	Yes	Excellent

7.	219Y1A0124	MOPURI CHENNAIAH	strongly agree	Good	Good	Yes	Excellent
8.	219Y1A0126	MURUKUTI MADHUSUDHAN REDDY	strongly agree	Good	Good	Yes	Excellent
9.	219Y1A0137	RACHAM REDDY PRAVEEN KUMAR REDDY	strongly agree	Good	Good	Yes	Excellent
10.	219Y1A0138	RAMAIAH GARI LOKESH	agree	Good	Good	Yes	Good
11.	219Y1A0139	RAMANABOINA CHAKRI	strongly agree	Good	Good	Yes	Excellent
12.	219Y1A0142	SEELAM STEEVAN	strongly agree	Good	Good	Yes	Excellent
13.	219Y1A0143	SHAIK KHAN MAHAMMAD SAIF	strongly agree	Good	Good	Yes	Excellent
14.	219Y1A0145	SHAIK MOHAMMED HUSSAIN	strongly agree	Good	Excellent	Yes	Excellent
15.	219Y1A0146	SHAIK YUSUF	strongly agree	Good	Good	Yes	Excellent
16.	219Y1A0157	VALLAMKONDU PRANAV RASHIK	strongly agree	Good	Good	Yes	Excellent
17.	229Y5A0102	B VAMSI KRISHNA	strongly agree	Good	Good	Yes	Excellent
18.	229Y5A0104	BOGAM BHAGYA LAKSHMI	agree	Good	Good	Yes	Excellent
19.	229Y5A0105	BOYA SAI SIRISHA	strongly agree	Good	Good	Yes	Excellent
20.	229Y5A0108	CHALLA AKHILA	strongly agree	Good	Excellent	Yes	Good

21.	229Y5A0109	CHEEPATI SHAIK WAJIDULLA	strongly agree	Good	Good	Yes	Excellent
22.	229Y5A0110	CHINTHA PAVAN KUMAR REDDY	strongly agree	Excellent	Good	Yes	Excellent
23.	229Y5A0111	CHUNCHULA RUCHITHA	strongly agree	Good	Good	Yes	Excellent
24.	229Y5A0112	CONDURU MADHAVA	strongly agree	Good	Good	Yes	Excellent
25.	229Y5A0113	DANDUBOINA HEMANTH KUMAR	strongly agree	Good	Excellent	Yes	Excellent
26.	229Y5A0116	DUDEKULA FARHAN BASHA	strongly agree	Good	Good	Yes	Excellent
27.	229Y5A0117	DUPATI BHAVANA	strongly agree	Good	Good	Yes	Excellent
28.	229Y5A0118	ERUKULA ABHIGNA	strongly agree	Good	Good	Yes	Excellent
29.	229Y5A0119	GOLLA GUNASEKHAR	strongly agree	Good	Good	Yes	Excellent
30.	229Y5A0121	GUDA VENKATESH	strongly agree	Good	Good	Yes	Excellent
31.	229Y5A0123	IMMUBAIGARI MAHAMMAD SHARIF	strongly agree	Good	Good	Yes	Excellent
32.	229Y5A0124	KAMBAM SAI KUMAR	strongly agree	Good	Good	Yes	Excellent
33.	229Y5A0130	LANKALA SUMITHRA	strongly agree	Excellent	Good	Yes	Excellent
34.	229Y5A0133	MALLEBOYINA SAI TEJA	strongly agree	Good	Good	Yes	Excellent

35.	229Y5A0136	MUDE MOHAN KRISHNA NAIK	strongly agree	Good	Good	Yes	Excellent
36.	229Y5A0137	NAKKALA AKHILA	strongly agree	Good	Good	Yes	Excellent
37.	229Y5A0138	P SAIKIRAN	strongly agree	Good	Good	Yes	Good
38.	229Y5A0140	RAJENDRAM JAYANTHACHAR	strongly agree	Good	Good	Yes	Excellent
39.	219Y1A0142	SEELAM STEEVAN	strongly agree	Excellent	Good	Yes	Excellent
40.	229Y5A0144	SASHI VARDHAN	agree	Good	Good	Yes	Excellent
41.	229Y5A0146	SHAIK ARIF	strongly agree	Good	Good	Yes	Excellent
42.	229Y5A0153	ULLASI RAGHAVENDRA	strongly agree	Good	Excellent	Yes	Excellent
43.	229Y5A0155	UPPARA VENKATESH	strongly agree	Good	Good	Yes	Excellent
44.	229Y5A0154	UPENDRAM VISHNU VARDHAN RAJU	strongly agree	Good	Good	Yes	Excellent
45.	229Y5A0157	YANABOTHULA PRASANNAKUMAR	strongly agree	Excellent	Good	Yes	Excellent
46.	229Y5A0159	YARRAMREDDY SIVAGANGADHAR REDDY	strongly agree	Good	Good	Yes	Excellent

G. Radha
Coordinator

HOD

Dr. I. SREEVANI M.Sc., Ph.D
Professor & HOD
Dept.of Humanities & sciences
K.S.R.M. College of Engineering
KADAPA Dist.

K.S.R.M. COLLEGE OF ENGINEERING (AUTONOMOUS), KADAPA-516003
DEPARTMENT OF HUMANITIES AND SCIENCES
CERTIFICATE COURSE ON
COMPLEX ANALYSIS FROM 28/11/2022 TO 03/01/2023
AWARD LIST

S. No.	Roll Number	Name of the Student	Marks Obtained
1.	219Y1A0101	ANAGONDI LAKSHMI NARASIMHA	15
2.	219Y1A0108	GAMPA UDAY KIRAN	18
3.	219Y1A0110	GUNDE SIDDARTHA NAIDU	19
4.	219Y1A0113	KANDLLI SIVA KUMAR	20
5.	219Y1A0115	KOMMALURU VENKATA SUJITH	20
6.	219Y1A0118	KOTHAPALLI AMARENDRANATH	16
7.	219Y1A0124	MOPURI CHENNAIAH	17
8.	219Y1A0126	MURUKUTI MADHUSUDHAN REDDY	15
9.	219Y1A0137	RACHAM REDDY PRAVEEN KUMAR REDDY	10
10.	219Y1A0138	RAMAIAH GARI LOKESH	11
11.	219Y1A0139	RAMANABOINA CHAKRI	18
12.	219Y1A0142	SEELAM STEEVAN	16
13.	219Y1A0143	SHAIK KHAN MAHAMMAD SAIF	12
14.	219Y1A0145	SHAIK MOHAMMED HUSSAIN	16
15.	219Y1A0146	SHAIK YUSUF	15
16.	219Y1A0157	VALLAMKONDU PRANAV RASHIK	14
17.	229Y5A0102	B VAMSI KRISHNA	17
18.	229Y5A0104	BOGAM BHAGYA LAKSHMI	19
19.	229Y5A0105	BOYA SAI SIRISHA	19
20.	229Y5A0108	CHALLA AKHILA	18
21.	229Y5A0109	CHEEPATI SHAIK WAJIDULLA	13
22.	229Y5A0110	CHINTHA PAVAN KUMAR REDDY	12
23.	229Y5A0111	CHUNCHULA RUCHITHA	11
24.	229Y5A0112	CONDURU MADHAVA	13
25.	229Y5A0113	DANDUBOINA HEMANTH KUMAR	12
26.	229Y5A0116	DUDEKULA FARHAN BASHA	10
27.	229Y5A0117	DUPATI BHAVANA	11
28.	229Y5A0118	ERUKULA ABHIGNA	12
29.	229Y5A0119	GOLLA GUNASEKHAR	13
30.	229Y5A0121	GUDA VENKATESH	14
31.	229Y5A0123	IMMUBAIGARI MAHAMMAD SHARIF	15
32.	229Y5A0124	KAMBAM SAI KUMAR	13
33.	229Y5A0130	LANKALA SUMITHRA	14
34.	229Y5A0133	MALLEBOYINA SAI TEJA	19
35.	229Y5A0136	MUDE MOHAN KRISHNA NAIK	18
36.	229Y5A0137	NAKKALA AKHILA	14
37.	229Y5A0138	P SAIKIRAN	13
38.	229Y5A0140	RAJENDRAM JAYANTHACHAR	13

39.	219Y1A0142	SEELAM STEEVAN	12
40.	229Y5A0144	SASHI VARDHAN	14
41.	229Y5A0146	SHAIK ARIF	12
42.	229Y5A0153	ULLASI RAGHAVENDRA	13
43.	229Y5A0155	UPPARA VENKATESH	12
44.	229Y5A0154	UPENDRAM VISHNU VARDHAN RAJU	12
45.	229Y5A0157	YANABOTHULA PRASANNAKUMAR	13
46.	229Y5A0159	YARRAMREDDY SIVAGANGADHAR REDDY	11

G. Radha
Coordinator

HoD

Dr. I. SREEVANI M.Sc.,Ph.D
Head of Humanities & Sciences
K.S.R.M. College of Engineering
KADAPA - 516 005

15/20

K.S.R.M. COLLEGE OF ENGINEERING (AUTONOMOUS), KADAPA-516003
DEPARTMENT OF HUMANITIES AND SCIENCES
CERTIFICATE COURSE ON
COMPLEX ANALYSIS FROM 28/11/2022 TO 03/01/2023
ASSESSMENT TEST

Roll Number: 219Y1A0101 **Name of the Student:** A. Lakshmi Narasimha

Time: 20 Min **(Objective Questions)** **Max.Marks: 20**

Note: Answer the following Questions and each question carries **one** mark.

1. If $f(z)=z^2$ for all z then [a] ✓
   a) f(z) is continuous at z = i
   b) f (z) is not continuous at z = i
   c) f (z) is continuous at z = 1
   d) None
2. If f(z) = z(2-z) then at z = 1+i, f(z) = [a] ✓
   a) 2 b) 0 c) i d) -i
3. The derivative of w = f(z) = z^3-2z at the point z = -1 [a] ✓
   a) 1 b) -1 c) I d) -i
4. If |sin z| ≤ 1 is true when [c] ✓
   a) for all z b) z is purely imaginary
   c) z is purely real d) none
5. The function f(z) = $\overline{z}$ is [c] ✓
   a) analytic at z = 0 b) analytic everywhere except at z = 0
   c) not analytic everywhere d) none
6. Cauchy-Riemann equations are: [a] ✓
   a) $u_x = v_y$ and $v_x = -u_y$ b) $u_x = v_y$, $v_x = u_y$
   c) $u_x = u_y$ and $v_x = v_y$ d) none
7. An analytic function with constant modulus is a [c] ✓
   a) function of x b) function of y
   c) constant function d) function of x and y
8. Functions which satisfy Laplace's equation in a region R is called_______ in R [a] ✓
   a) harmonics b) non harmonic
   c) analytic d) none
9. A point at which f(z) fails to be analytic is called ____ of f(z). [a] ✓
   a) singular point b) zero point
   c) null point d) none
10. The value of $\int_c \frac{z}{z^2+1} dz$ where c is |z+i| = $\frac{1}{2}$ is [b] ✓
    a) 0 b) πi c) πi d) 2πi
11. The value of $\int_c \frac{e^{iz}}{z+3i} dz$ where c is the circle |z+3i| = 1 is [d] ✓
    a) 0 b) 2πi c) $2\pi i.e^2$ d) $2\pi i.e^3$

12. The function f(z) = z|z| is [b]

a) analytic everywhere
b) non analytic everywhere
c) analytic for all finite values of z except at z=0
d) none of this

13. The function f(z) = xy + iy is [a]

a) everywhere continues but not analytic
b) discontinues but analytic everywhere
c) everywhere continuous and analytic
d) neither continues nor analytic

14. The analytic function among the following is [d]

a) f(z) = Re(z)
b) f(z) =Im(z)
c) $f(z) = \overline{z}$
d) f(z) = sinz

15. The function $f(z) = |z|^2$ is [b]

a) differentiable anywhere
b) differentiable only at the origin
c) not differentiable anywhere
d) none of this

16. If f(z) be analytic within and on a simple closed contour c then the point giving the maximum of |f(z)| can be [d]

a) within c
b) outside c
c) on the boundary c and not with in it
d) on the boundary and within c

17. The value of $\int_c \frac{dz}{z}$,where c is |z| = r is [d]

a) πi
b) $\frac{\pi i}{2}$
c) 2πi
d) log r

18. A continuous arc without multiple points is called a [d]

a) continuous arc
b) contour
c) Jordan arc
d) none of this

19. A point at which a function f(z) ceases to be analytic is called [b]

a) zero
b) infinity
c) anywhere
d) curve c

20. At z=0 ,$\tan\frac{1}{z}$ has [a]

a) a simple pole
b) a double pole
d) an isolated singularity
d) a non-isolated essential singularity

14/20

K.S.R.M. COLLEGE OF ENGINEERING (AUTONOMOUS), KADAPA-516003
DEPARTMENT OF HUMANITIES AND SCIENCES
CERTIFICATE COURSE ON
COMPLEX ANALYSIS FROM 28/11/2022 TO 03/01/2023
ASSESSMENT TEST

Roll Number: 229Y5A0121 **Name of the Student:** G. Venkatesh.

Time: 20 Min **(Objective Questions)** **Max.Marks: 20**

Note: Answer the following Questions and each question carries **one** mark.

1. If $f(z)=z^2$ for all z then [a] ✓
   a) f(z) is continuous at z = i
   b) f (z) is not continuous at z = i
   c) f (z) is continuous at z = 1
   d) None
2. If f(z) = z(2-z) then at z = 1+i,f(z) = [a] ✓
   a) 2 b) 0 c) i d) -i
3. The derivative of $w = f(z) = z^3-2z$ at the point z = -1 [a] ✓
   a) 1 b) -1 c) I d) -i
4. If $|\sin z| \leq 1$ is true when [c] ✓
   a) for all z b) z is purely imaginary
   c) z is purely real d) none
5. The function $f(z) = \bar{z}$ is [a] ✗
   a) analytic at z = 0 b) analytic everywhere except at z = 0
   c) not analytic everywhere d) none
6. Cauchy-Riemann equations are: [a] ✓
   a) $u_x = v_y$ and $v_x = -u_y$ b) $u_x = v_y$, $v_x = u_y$
   c) $u_x = u_y$ and $v_x = v_y$ d) none
7. An analytic function with constant modulus is a [c] ✓
   a) function of x b) function of y
   c) constant function d) function of x and y
8. Functions which satisfy Laplace's equation in a region R is called_______ in R [a] ✓
   a) harmonics b) non harmonic
   c) analytic d) none
9. A point at which f(z) fails to be analytic is called ____ of f(z). [a] ✓
   a) singular point b) zero point
   c) null point d) none
10. The value of $\int_c \frac{z}{z^2+1} dz$ where c is $|z+i| = \frac{1}{2}$ is [d] ✗
    a) 0 b) πi c) πi d) 2πi
11. The value of $\int_c \frac{e^{iz}}{z+3i} dz$ where c is the circle $|z+3i| = 1$ is [d] ✓
    a) 0 b) 2πi c) $2\pi i.e^2$ d) $2\pi i.e^3$

12. The function f(z) = z|z| is [b]
   a) analytic everywhere   b) non analytic everywhere
   c) analytic for all finite values of z except at z=0   d) none of this
13. The function f(z) = xy + iy is [a]
   a) everywhere continues but not analytic
   b) discontinues but analytic everywhere
   c) everywhere continuous and analytic
   d) neither continues nor analytic
14. The analytic function among the following is [a]
   a) f(z) = Re(z)   b) f(z) =Im(z)
   c) $f(z) = \overline{z}$ d) f(z) = sinz
15. The function $f(z) = |z|^2$ is [b]
   a) differentiable anywhere   b) differentiable only at the origin
   c) not differentiable anywhere   d) none of this
16. If f(z) be analytic within and on a simple closed contour c then the point giving the maximum of |f(z)| can be [a]
   a) within c   b) outside c
   c) on the boundary c and not with in it   d) on the boundary and within c
17. The value of $\int_C \frac{dz}{z}$,where c is |z| = r is [d]
   a) πi b) $\frac{\pi i}{2}$ c) $2\pi i$ d) $\log r$
18. A continuous arc without multiple points is called a [b]
   a) continuous arc   b) contour   c) Jordan arc   d) none of this
19. A point at which a function f(z) ceases to be analytic is called [d]
   a) zero   b) infinity   c) anywhere   d) curve c
20. At z=0 ,$\tan\frac{1}{z}$ has [a]
   a) a simple pole   b) a double pole
   d) an isolated singularity   d) a non-isolated essential singularity

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K.S.R.M. COLLEGE OF ENGINEERING (AUTONOMOUS), KADAPA-516003
DEPARTMENT OF HUMANITIES AND SCIENCES
CERTIFICATE COURSE ON
COMPLEX ANALYSIS FROM 28/11/2022 TO 03/01/2023
ASSESSMENT TEST

Roll Number: 229Y1A0119 **Name of the Student:** Golla Gunasekhar

Time: 20 Min **(Objective Questions)** **Max.Marks: 20**

Note: Answer the following Questions and each question carries **one** mark.

1. If $f(z)=z^2$ for all z then [a]
   a) f(z) is continuous at z = i
   b) f (z) is not continuous at z = i
   c) f (z) is continuous at z = 1
   d) None
2. If f(z) = z(2-z) then at z = 1+i,f(z) = [b]
   a) 2 b) 0 c) i d) -i
3. The derivative of w = f(z) = z^3-2z at the point z = -1 [c]
   a) 1 b) -1 c) I d) -i
4. If |sin z| ≤ 1 is true when [d]
   a) for all z b) z is purely imaginary
   c) z is purely real d) none
5. The function f(z) = $\bar{z}$ is [a]
   a) analytic at z = 0 b) analytic everywhere except at z = 0
   c) not analytic everywhere d) none
6. Cauchy-Riemann equations are: [a]
   a) $u_x = v_y$ and $v_x = -u_y$ b) $u_x = v_y$, $v_x = u_y$
   c) $u_x = u_y$ and $v_x = v_y$ d) none
7. An analytic function with constant modulus is a [c]
   a) function of x b) function of y
   c) constant function d) function of x and y
8. Functions which satisfy Laplace's equation in a region R is called_______ in R [a]
   a) harmonics b) non harmonic
   c) analytic d) none
9. A point at which f(z) fails to be analytic is called ____ of f(z). [a]
   a) singular point b) zero point
   c) null point d) none
10. The value of $\int_c \frac{z}{z^2+1} dz$ where c is $|z+i| = \frac{1}{2}$ is [b]
    a) 0 b) πi c) πi d) 2πi
11. The value of $\int_c \frac{e^{iz}}{z+3i} dz$ where c is the circle |z+3i| = 1 is [b]
    a) 0 b) 2πi c) 2πi.e^2 d) 2πi.e^3

12. The function f(z) = z|z| is [b]
a) analytic everywhere b) non analytic everywhere
c) analytic for all finite values of z except at z=0 d) none of this

13. The function f(z) = xy + iy is [a]
a) everywhere continues but not analytic
b) discontinues but analytic everywhere
c) everywhere continuous and analytic
d) neither continues nor analytic

14. The analytic function among the following is [d]
a) f(z) = Re(z) b) f(z) =Im(z)
c) f(z) = $\bar{z}$ d) f(z) = sinz

15. The function f(z) = $|z|^2$ is [b]
a) differentiable anywhere b) differentiable only at the origin
c) not differentiable anywhere d) none of this

16. If f(z) be analytic within and on a simple closed contour c then the point giving the maximum of |f(z)| can be [d]
a) within c b) outside c
c) on the boundary c and not with in it d) on the boundary and within c

17. The value of $\int_C \frac{dz}{z}$,where c is |z| = r is [d]
a) πi b) $\frac{\pi i}{2}$ c) 2πi d) log r

18. A continuous arc without multiple points is called a [C]
a) continuous arc b) contour c) Jordan arc d) none of this

19. A point at which a function f(z) ceases to be analytic is called [d]
a) zero b) infinity c) anywhere d) curve c

20. At z=0 ,tan $\frac{1}{z}$ has [d]
a) a simple pole b) a double pole
d) an isolated singularity d) a non-isolated essential singularity

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K.S.R.M. COLLEGE OF ENGINEERING (AUTONOMOUS), KADAPA-516003
DEPARTMENT OF HUMANITIES AND SCIENCES
CERTIFICATE COURSE ON
COMPLEX ANALYSIS FROM 28/11/2022 TO 03/01/2023
ASSESSMENT TEST

Roll Number: 219Y1A0146 **Name of the Student:** Shaik Yusuf

Time: 20 Min **(Objective Questions)** **Max.Marks: 20**

Note: Answer the following Questions and each question carries **one** mark.

1. If $f(z)=z^2$ for all z then [a]
   a) f(z) is continuous at z = i
   b) f (z) is not continuous at z = i
   c) f (z) is continuous at z = 1
   d) None
2. If f(z) = z(2-z) then at z = 1+i,f(z) = [a]
   a) 2 b) 0 c) i d) -i
3. The derivative of w = f(z) = z^3-2z at the point z = -1 [a]
   a) 1 b) -1 c) I d) -i
4. If |sin z| ≤ 1 is true when [c]
   a) for all z b) z is purely imaginary
   c) z is purely real d) none
5. The function f(z) = $\overline{z}$ is [C]
   a) analytic at z = 0 b) analytic everywhere except at z = 0
   c) not analytic everywhere d) none
6. Cauchy-Riemann equations are: [a]
   a) $u_x = v_y$ and $v_x = -u_y$ b) $u_x = v_y$, $v_x = u_y$
   c) $u_x = u_y$ and $v_x = v_y$ d) none
7. An analytic function with constant modulus is a [c]
   a) function of x b) function of y
   c) constant function d) function of x and y
8. Functions which satisfy Laplace's equation in a region R is called_______ in R [a]
   a) harmonics b) non harmonic
   c) analytic d) none
9. A point at which f(z) fails to be analytic is called ____ of f(z). [a]
   a) singular point b) zero point
   c) null point d) none
10. The value of $\int_c \frac{z}{z^2+1} dz$ where c is $|z+i| = \frac{1}{2}$ is [b]
    a) 0 b) πi c) πi d) 2πi
11. The value of $\int_c \frac{e^{iz}}{z+3i} dz$ where c is the circle |z+3i| = 1 is [d]
    a) 0 b) 2πi c) 2πi.e^2 d) 2πi.e^3

12. The function $f(z) = z|z|$ is [b]

a) analytic everywhere
b) non analytic everywhere
c) analytic for all finite values of z except at z=0
d) none of this

13. The function $f(z) = xy + iy$ is [a]

a) everywhere continues but not analytic
b) discontinues but analytic everywhere
c) everywhere continuous and analytic
d) neither continues nor analytic

14. The analytic function among the following is [d]

a) $f(z) = Re(z)$
b) $f(z) = Im(z)$
c) $f(z) = \bar{z}$
d) $f(z) = \sin z$

15. The function $f(z) = |z|^2$ is [b]

a) differentiable anywhere
b) differentiable only at the origin
c) not differentiable anywhere
d) none of this

16. If f(z) be analytic within and on a simple closed contour c then the point giving the maximum of $|f(z)|$ can be [d]

a) within c
b) outside c
c) on the boundary c and not with in it
d) on the boundary and within c

17. The value of $\int_c \frac{dz}{z}$,where c is $|z| = r$ is [d]

a) πi
b) $\frac{\pi i}{2}$
c) $2\pi i$
d) $\log r$

18. A continuous arc without multiple points is called a [c]

a) continuous arc
b) contour
c) Jordan arc
d) none of this

19. A point at which a function f(z) ceases to be analytic is called [b]

a) zero
b) infinity
c) anywhere
d) curve c

20. At z=0 ,$\tan\frac{1}{z}$ has [a]

a) a simple pole
b) a double pole
d) an isolated singularity
d) a non-isolated essential singularity

Module 1

Function of a Complex Variables:

If $z = x + iy$ and $w = u + iv$ are two complex variables, and if for each value of z in a certain portion of the complex plane (called also as the domain R of the complex plane) there corresponds one or more values of y, then w is said to be a function of z and is written as

$$w = f(z) = f(x + iy) = u(x, y) + i\, v(x, y) \qquad (1)$$

where $u(x, y)$and $v(x, y)$ are real functions of the real variables x and y. Clearly for a given value of z, the values of x and y are known and thus, one or more values of w are determined by (1). If for each value of z in R, there is correspondingly only one value of w, then w is called a *single-valued function* of z. If there is more than one value of w corresponding to a given value of z, then w is called a *multiple-valued function or many- valued function.*

For example, $w = z^2, w = \frac{1}{z}, w = \frac{z}{z^4+1}$ are single valued function of z. The function $w = z^{1/2}$, $w = \arg(z)$ are examples of many valued functions. The first one has three values for each value of z (except for $z = 0$) and the second one assumes infinite set of real values for each value of z other than $z = 0$.

The complex quantities z and w can be represented on separate complex planes, called the z-plane and the w-plane respectively. The relation $w = f(z)$ establishes correspondence between the points (x, y) of the z-plane and the points (u, v) of the w-plane.

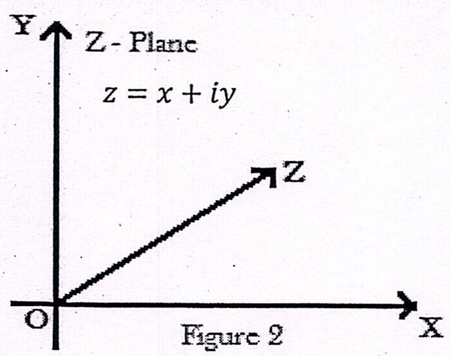

Figure 2

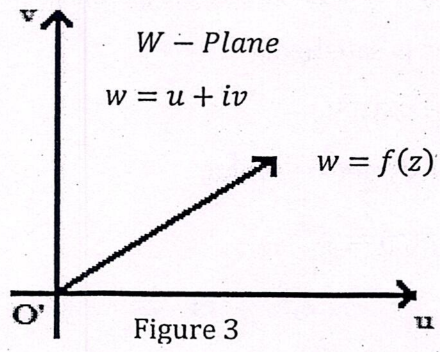

Figure 3

Limits: Let $w = f(z)$ denote some functional relationship connecting w with z.

Then $w = f(x+iy) = u(x,y) + i\,v(x,y)$ where u and v are real functions of x and y. As z approaches z_0, the limit of $f(z)$ is said to be w_0 if $f(z)$ can be kept arbitrarily close to w_0, by keeping z sufficiently close to, but different from z_0.

$$i.e., \lim_{z\to z_0} w = \lim_{z\to z_0} f(z) = w_0$$

Now let $z_0 = x_0 + iy_0$

when z approaches z_0, it means that $x \to x_0$ and $y \to y_0$.

Hence $\lim_{z\to z_0} f(z) = \lim_{z\to z_0} (u + i\,v) = \lim_{\substack{x\to x_0 \\ y\to y_0}} (u + i\,v) = u_0 + i\,v_0$

Hence $\lim_{\substack{x\to x_0 \\ y\to y_0}} u(x,y) = u_0$ and $\lim_{\substack{x\to x_0 \\ y\to y_0}} v(x,y) = v_0$.

Note: In the above, when we say that $z \to z_0$, it means that $x \to x_0$ *and* $y \to y_0$ in any order, by any path as shown in figure 4.

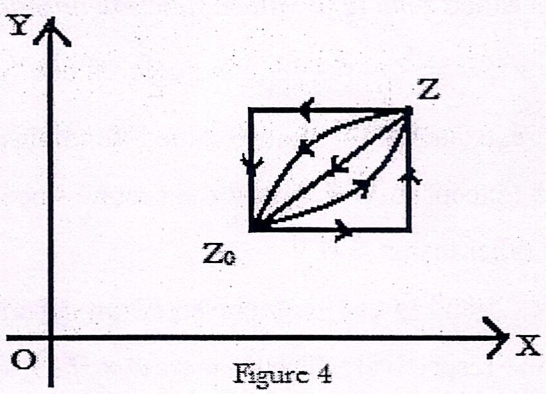

Figure 4

Continuity: The idea of continuity is closely connected with the concept of a limit. A single-valued function $w = f(z)$ is said to be continuous at a point $z = z_0$ provided each of the following conditions is satisfied:

(i) $f(z_0)$ exists

(ii) $\lim_{z\to z_0} f(z)$ exists, and

(iii) $\lim_{z\to z_0} f(z) = f(z_0)$

Remarks:

1. If $f(z)$ is continuous at every point of a region R, it is said to be continuous throughout R.

2. $w = f(z) = u(x,y) + i\,v(x,y)$. If $f(z)$is continuous at $z = z_0$, then its real and imaginary parts, i.e., u and v will be continuous functions at $z = z_0$, i.e., at $x = x_0$ *and* $y =$

y_0. Conversely, if u and v are continuous functions at $z = z_0$, then $f(z)$ will be continuous at $z = z_0$.

3. The sums, differences and products of continuous functions are also continuous are also continuous. The quotient of two continuous functions is continuous except for those values of z for which the denominator vanishes.

Continuity of a Function of Two Real Variables:

$$w = f(z) = f(x + iy)$$

is a function of the two variables x and y. Hence, to discuss the continuity of $f(z)$, we shall have to deal with the continuity of a function of two independent variables x and y.

Definition: a function $f(x,\ y)$ of two real independent variables x and y is said to be continuous at a point $(x_0,\ y_0)$ if,

(i) $f(x_0,\ y_0)$, the value of $f(x,\ y)$ at $(x_0,\ y_0)$ is finite, and

(ii) $\lim\limits_{\substack{x\to x_0 \\ y\to y_0}} f(x,y) = f(x_0,\ y_0)$ in whatever way $x \to x_0$ *and* $y \to y_0$

To illustrate the idea of continuity of a function of two variables given in the following examples:

EX. 1. Show that $f(x,y) = \frac{2xy}{x^2+y^2}$ is discontinuous at origin, given that $f(0,0) = 0$.

Solution: Given $f(x,y) = \frac{2xy}{x^2+y^2}$

If $y \to 0$ first and then $x \to 0$

$$\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{x\to 0}\lim_{y\to 0}\frac{2xy}{x^2 + y^2} = \lim_{x\to 0}\frac{2x(0)}{x^2} = 0$$

If $x \to 0$ first and then $y \to 0$

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0}\frac{2xy}{x^2 + y^2} = \lim_{y\to 0}\frac{2y(0)}{y^2} = 0$$

Let x and y both tend to zero simultaneously along the path $y = mx$.

$$\text{Then, } \lim_{\substack{y=mx \\ x\to 0}} f(x,y) = \lim_{\substack{y=mx \\ x\to 0}}\frac{2xy}{x^2 + y^2} = \lim_{x\to 0}\frac{2x.mx}{x^2 + m^2x^2} = \frac{2m}{1 + m^2}$$

This limit changes its value for different values of m.

when $m = 1, \frac{2m}{1+m^2} = 1$ and for $m = 2, \frac{2m}{1+m^2} = \frac{4}{5}$ and so on.

Hence $\lim\limits_{y\to 0}\frac{2xy}{x^2+y^2} \neq 0$, when $x \to 0, y \to 0$ in any manner. So the function is not continuous at the origin.

Derivative of a Function of a Complex Variable: For a real function of a single real variable say, $y = f(x)$, the derivative of y with Respect to x is defined as

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Hence Δx can approach zero in only one way.

Let $w = f(z)$ be a single-valued function of z. Then, the derivative of w is defined to be

$$\frac{dw}{dz} = f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

provided the above limit exists and is the same, in whatever manner Δz approaches zero.

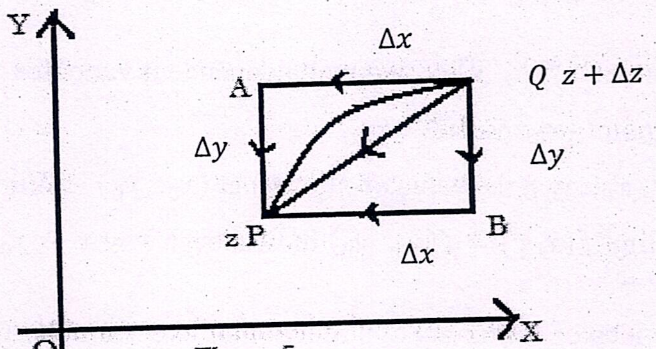

Figure 5

We can show by a figure that Δz can approach zero in several ways. P is the point in the z-plane corresponding to $z = x + i\,y$. Q is the point $z + \Delta z$. $\Delta z = \Delta x + i\,\Delta y$, where $\Delta x, \Delta y$ are small increments in x *and* y respectively. As $\Delta z \to 0$, *i.e.*, $\Delta x, \Delta y$ *also* $\to 0$ and the point Q approaches to P. Now Q can approach P along the rectilinear path QAP on which first Δx and then Δy approach zero or Q may approach P along the rectilinear path QBP on which first Δy and then Δx approach zero. More generally, Q can approach P along infinitely many paths, *i.e.*, Δz approaches zero in several ways.

Hence, in the definition of $f'(z)$, the derivative of $f(z)$, it is necessary that the limit of the difference quotient

$$i.e., \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

should be the same, no matter how Δz approaches zero. When this limit is unique, the function is said to be differentiable. This severe restriction narrows down greatly the class of functions of a complex variable that possess derivatives.

Thus we find that $\frac{dw}{dz}$ depends not only upon z but also upon the manner in which Δz approaches zero. To illustrate this, consider the simple case,

$$w = f(z) = x - i\,y$$

Then

$$\frac{f(z+\Delta z) - f(z)}{\Delta z} = \frac{[(x+\Delta x) - i\,(y+\Delta y)] - (x - i\,y)}{\Delta x + i\,\Delta y}$$

$$= \frac{\Delta x - i\,\Delta y}{\Delta x + i\,\Delta y}$$

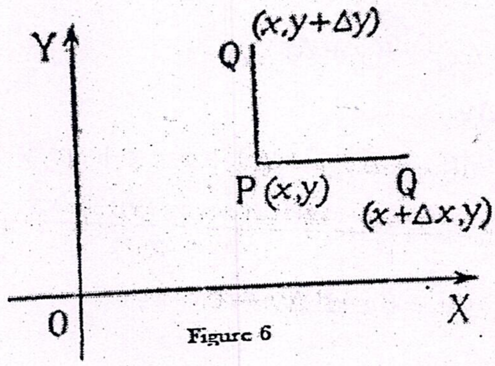

Figure 6

Now, let $\Delta z \to 0$ is such a way that first Δy and then Δx approach zero, i.e., Q approaches P along the horizontal line. Then

$$\lim_{\Delta z \to 0} \frac{\Delta x - i\,\Delta y}{\Delta x + i\,\Delta y} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = 1$$

But, suppose Q approaches P along the vertical line so that first Δx and then Δy approach zero. Then

$$\lim_{\Delta z \to 0} \frac{\Delta x - i\,\Delta y}{\Delta x + i\,\Delta y} = \lim_{\Delta y \to 0} \frac{-i\,\Delta y}{i\,\Delta y} = -1$$

For other paths of approach of Q towards P, we can get as many distinct values of the above limit as we please. We therefore say that $f(z) = x - i\,y$ possesses no derivative.

Definition: If a single-valued function $w = f(z)$ possesses a derivative at $z = z_0$ and at every point in some neighbourhood of z_0, then $f(z)$ is said to be *analytic* at z_0 and z_0 is called a *regular point* of the function. If $f(z)$ is analytic at every point of a region R, then we say that $f(z)$is analytic in R. A point at which an analytic function ceases to have a derivative is called a singular point. An analytic function is also referred to as *regular* or *holomorphic.*

Conditions under which $w = f(z)$ is analytic:

Let $w = f(z)$ be an analytic function of a complex variable in a region R. Then $f'(z)$ exists at every point in R. Let us now find the conditions for the existence of the derivative of $f(z)$ at a point z.

Let $z = x + i\,y$ and $w = f(z) = f(x + i\,y) = u(x,y) + i\,v(x,y)$ where u and v are functions of x and y. Let Δx and Δy be the increments in x and y respectively and let Δz be the corresponding increment in z

Then $z + \Delta z = (x + \Delta x) + i(y + \Delta y)$

Hence $\Delta z = \Delta x + i\,\Delta y$

Also $f(z + \Delta z) = u(x + \Delta x, y + \Delta y) + i\,v(x + \Delta x, y + \Delta y)$

Hence $\frac{f(z+\Delta z)-f(z)}{\Delta z} = \frac{[u(x+\Delta x,y+\Delta y)+i\,v(x+\Delta x,y+\Delta y)]-[u(x,y)+i\,v(x,y)]}{\Delta x+i\,\Delta y}$

As $\Delta z \to 0$, we have $\Delta x \to 0$ and $\Delta y \to 0$.

Hence by definition,

$$f'(z) = \lim_{\Delta z\to 0}\frac{f(z+\Delta z)-f(z)}{\Delta z}$$

$$f'(z) = \lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\frac{[u(x+\Delta x,y+\Delta y)+i\,v(x+\Delta x,y+\Delta y)]-[u(x,y)+i\,v(x,y)]}{\Delta x+i\,\Delta y} \qquad (1)$$

If $f(z)$ is analytic, $f'(z)$ must have a unique value, in whatever manner $\Delta z \to 0$. Now let $\Delta z \to 0$ in such a way that first Δy and then $\Delta x \to 0$. Then from (1),

$$f'(z) = \lim_{\Delta x\to 0}\frac{[u(x+\Delta x,y)+i\,v(x+\Delta x,y)]-[u(x,y)+i\,v(x,y)]}{\Delta x}$$

$$i.e., f'(z) = \lim_{\Delta x\to 0}\frac{[u(x+\Delta x,y)-u(x,y)]+i\,[v(x+\Delta x,y)-v(x,y)]}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{u(x+\Delta x,y)-u(x,y)}{\Delta x} + i\lim_{\Delta x\to 0}\frac{v(x+\Delta x,y)-v(x,y)}{\Delta x}$$

$$= \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \qquad (2)$$

(by definition of partial derivatives)

Since $f'(z)$ is to be unique, it is necessary that the partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$ must exists at the point (x,y).

Secondly, let $\Delta z \to 0$ such that $\Delta x \to 0$ first and then $\Delta y \to 0$. Then from (1)

$$f'(z) = \lim_{\Delta y\to 0}\frac{[u(x,y+\Delta y)+i\,v(x,y+\Delta y)]-[u(x,y)+i\,v(x,y)]}{i\,\Delta y}$$

$$i.e., f'(z) = \lim_{\Delta y \to 0} \frac{[u(x, y + \Delta y) - u(x, y)] + i\,[v(x, y + \Delta y) - v(x, y)]}{i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(x, y + \Delta y) - u(x, y)}{i\,\Delta y} + \lim_{\Delta y \to 0} \frac{v(x, y + \Delta y) - v(x, y)}{\Delta y}$$

$$= \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = \frac{\partial v}{\partial y} - i\,\frac{\partial u}{\partial y} \qquad (3)$$

Hence $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial y}$ must exist at (x, y).

Now, if the derivative $f'(z)$ exists, it is necessary that the two expressions (2) and (3) which we have derived for it must be the same. Hence equating these expressions, we have

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\,\frac{\partial u}{\partial y}$$

Equating real and imaginary parts, we get

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad (4)$$

$$\text{and } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \qquad (5)$$

$$i.e., u_x = v_y \text{ and } v_x = -u_y$$

The equations (4) and (5) are called ***Cauchy-Riemann differential equations***.

Note: The Cauchy-Riemann equations are only the necessary conditions for the function $f(z) = u + i\,v$ to be differentiable i.e., if the function is differentiable, then it must satisfy these equations. But the converse is not necessarily true. A function may satisfy these equations at a point and yet it may not be differentiable at that point.

Hence the conditions expressed by Cauchy-Riemann equations (C-R equations) are only *necessary but not sufficient* for a function to be analytic.

Sufficient Conditions for $f(z)$ to be Analytic: We shall now prove the following theorem

The single valued continuous function $w = f(z) = u + i\,v$ analytic in a region R, if the four partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$ *and* $\frac{\partial v}{\partial y}$ exist, are continuous and satisfy the ***Cauchy-Riemann equations*** at each point in$R$.

Proof: Let $w = f(z) = u(x, y) + i\,v(x, y)$

It is now given that

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}\ and\ \frac{\partial v}{\partial x}=-\frac{\partial u}{\partial y} \tag{1}$$

Also these partial derivatives are continuous.

Then $\Delta u = u(x+\Delta x, y+\Delta y) - u(x,y)$

$$= [u(x+\Delta x, y+\Delta y) - u(x+\Delta x, y)] + [u(x+\Delta x, y) - u(x,y)]$$

$$= \Delta y.\frac{\partial}{\partial y}u(x+\Delta x, y+\theta_1.\Delta y) + \Delta x.\frac{\partial}{\partial x}u(x+\theta_2.\Delta x, y)$$

Using the first Mean Value Theorem, θ_1 and θ_2 being both positive and less than 1.

Now, at the point (x,y) the derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ are continuous.

Hence the above expression Δu may be written as

$$\Delta u = \Delta x.\left[\frac{\partial u}{\partial x}+\lambda_1\right] + \Delta y.\left[\frac{\partial u}{\partial y}+\lambda_2\right] \tag{2}$$

where λ_1 and λ_2 both tend to zero as $|\Delta z| \to 0$

Similarly, using the result that the derivatives $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ are continuous, we get

$$\Delta v = \Delta x.\left[\frac{\partial v}{\partial x}+\mu_1\right] + \Delta y.\left[\frac{\partial v}{\partial y}+\mu_2\right] \tag{3}$$

where μ_1 and μ_2 both tend to zero as $|\Delta z| \to 0$

Now $\Delta w = \Delta u + i\,\Delta v$

$$= \left\{\Delta x.\left[\frac{\partial u}{\partial x}+\lambda_1\right] + \Delta y.\left[\frac{\partial u}{\partial y}+\lambda_2\right]\right\} + i\left\{\Delta x.\left[\frac{\partial v}{\partial x}+\mu_1\right] + \Delta y.\left[\frac{\partial v}{\partial y}+\mu_2\right]\right\}$$

$$= \Delta x\left(\frac{\partial u}{\partial x}+i\,\frac{\partial v}{\partial x}\right) + \Delta y\left(\frac{\partial u}{\partial y}+i\,\frac{\partial v}{\partial y}\right) + \varepsilon_1\,\Delta x + \varepsilon_2\,\Delta y \tag{4}$$

where $\varepsilon_1 = \lambda_1 + i\,\mu_1$ and $\varepsilon_2 = \lambda_2 + i\,\mu_2$ and $\varepsilon_1, \varepsilon_2 \to 0$ as $|\Delta z| \to 0$.

In (4), apply the conditions (1) *i.e.*, put

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x}=-\frac{\partial u}{\partial y}$$

Then $\Delta w = \Delta x\left(\frac{\partial u}{\partial x}+i\,\frac{\partial v}{\partial x}\right) + \Delta y\left(-\frac{\partial v}{\partial x}+i\,\frac{\partial u}{\partial x}\right) + \varepsilon_1\,\Delta x + \varepsilon_2\,\Delta y$

$$= (\Delta x + i\,\Delta y)\frac{\partial u}{\partial x} + i\,(\Delta x + i\,\Delta y)\frac{\partial v}{\partial x} + \varepsilon_1\,\Delta x + \varepsilon_2\,\Delta y$$

$$= (\Delta x + i\,\Delta y)\left[\frac{\partial u}{\partial x}+i\,\frac{\partial v}{\partial x}\right] + \varepsilon_1\,\Delta x + \varepsilon_2\,\Delta y$$

Hence $$\frac{\Delta w}{\Delta z} = \frac{\partial u}{\partial x} + i\,\frac{\partial v}{\partial x} + \varepsilon_1\frac{\Delta x}{\Delta z} + \varepsilon_2\frac{\Delta y}{\Delta z} \tag{5}$$

By definition, $$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{[u(r+\Delta r, \theta+\Delta\theta) + i\,v(r+\Delta r, \theta+\Delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]}{\Delta(r\,e^{i\theta})} \qquad (1)$$

If $f(z)$ is analytic, $f'(z)$ must have a unique value in whatever manner $\Delta z \to 0$.

First let $\Delta z \to 0$ along a radius vector through the origin.

i.e., keep θ constant.

Then $\Delta z = \Delta(r\,e^{i\theta}) = e^{i\theta}\Delta r$.

As $\Delta z \to 0$, $\Delta r \to 0$. So (1) gives

$$f'(z) = \lim_{\Delta r \to 0} \frac{[u(r+\Delta r, \theta) + i\,v(r+\Delta r, \theta)] - [u(r,\theta) + i\,v(r,\theta)]}{e^{i\theta}\Delta r}$$

$$= e^{-i\theta} \lim_{\Delta r \to 0} \left[\frac{u(r+\Delta r,\theta) - u(r,\theta)}{\Delta r} + i\,\frac{v(r+\Delta r,\theta) - v(r,\theta)}{\Delta r}\right]$$

$$= e^{-i\theta} \left[\lim_{\Delta r \to 0} \frac{u(r+\Delta r,\theta) - u(r,\theta)}{\Delta r} + i\,\lim_{\Delta r \to 0} \frac{v(r+\Delta r,\theta) - v(r,\theta)}{\Delta r}\right]$$

$$= e^{-i\theta}\left(\frac{\partial u}{\partial r} + i\,\frac{\partial v}{\partial r}\right) \qquad (2)$$

Secondly, keep r constant.

Then $\Delta z = \Delta(r\,e^{i\theta}) = ire^{i\theta}\Delta\theta$

As $\Delta z \to 0$, $\Delta\theta \to 0$. So (1) gives

$$f'(z) = \lim_{\Delta\theta \to 0} \frac{[u(r, \theta+\Delta\theta) + i\,v(r, \theta+\Delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]}{ire^{i\theta}\Delta\theta}$$

$$= \frac{1}{re^{i\theta}} \lim_{\Delta\theta \to 0} \frac{[u(r, \theta+\Delta\theta) + i\,v(r, \theta+\Delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]}{i\Delta\theta}$$

$$= \frac{1}{re^{i\theta}} \lim_{\Delta\theta \to 0} \frac{[u(r, \theta+\Delta\theta) - u(r,\theta)] + i[\,v(r, \theta+\Delta\theta) - \,v(r,\theta)]}{i\Delta\theta}$$

$$= \frac{1}{re^{i\theta}}\left[-i\,\lim_{\Delta\theta \to 0} \frac{u(r,\theta+\Delta\theta) - u(r,\theta)}{\Delta\theta} + \lim_{\Delta\theta \to 0} \frac{v(r,\theta+\Delta\theta) - v(r,\theta)}{\Delta\theta}\right]$$

$$= \frac{1}{r}e^{-i\theta}\left(-i\,\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial\theta}\right) \qquad (3)$$

Now $|\Delta x| \leq |\Delta z|$ and $|\Delta y| \leq |\Delta z|$

and so $\left|\frac{\Delta x}{\Delta z}\right| \leq 1$ and $\left|\frac{\Delta y}{\Delta z}\right| \leq 1$.

Also $\varepsilon_1, \varepsilon_2 \to 0$ as $|\Delta z| \to 0$

So proceeding to the limit as $\Delta z \to 0$, (5) gives

$$\frac{dw}{dz} = \frac{\partial u}{\partial x} + i\,\frac{\partial v}{\partial x}$$

$i.e., f'(z)$ exists and is equal to $\frac{\partial u}{\partial x} + i\,\frac{\partial v}{\partial x}$

We shall put the above discussion in 4.7 and 4.8 relating to differentiability in the form of a theorem as follows.

If u and v are real single-valued functions of x and y which, with their four first order partial derivatives$\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}\right.$ and $\left.\frac{\partial v}{\partial y}\right)$, are continuous throughout a region R, then the

Cauchy-Riemann equations

$$u_x = v_y \text{ and } v_x = -u_y$$

are both necessary and sufficient condition, so that $f(z) = u + i\,v$ may be analytic. The derivative of $f(z)$ is then given by either of the expressions

$$f'(z) = \frac{\partial u}{\partial x} + i\,\frac{\partial v}{\partial x} \text{ or } f'(z) = \frac{\partial v}{\partial y} - i\,\frac{\partial u}{\partial y}$$

Derive the *Cauchy-Riemann equations* if $f(z)$ is expressed in polar coordinates.

Solution: Let $f(z) = u(r,\theta) + i\,v(r,\theta)$ in polar coordinates.

$z = x + i\,y = r(\cos\theta + i\,\sin\theta) = r\,e^{i\theta}$.

Let Δr and $\Delta\theta$ be the increments in r and θ respectively and let Δz be the corresponding increment in z.

$$\Delta z = \Delta\left(r\,e^{i\theta}\right)$$

$$f(z + \Delta z) = u(r + \Delta r, \theta + \Delta\theta) + i\,v(r + \Delta r, \theta + \Delta\theta)$$

$$f(z + \Delta z) - f(z) = [u(r + \Delta r, \theta + \Delta\theta) + i\,v(r + \Delta r, \theta + \Delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]$$

Hence $\dfrac{f(z + \Delta z) - f(z)}{\Delta z}$

$$= \frac{[u(r + \Delta r, \theta + \Delta\theta) + i\,v(r + \Delta r, \theta + \Delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]}{\Delta z}$$

$$= \frac{[u(r + \Delta r, \theta + \Delta\theta) + i\,v(r + \Delta r, \theta + \Delta\theta)] - [u(r,\theta) + i\,v(r,\theta)]}{\Delta(r\,e^{i\theta})}$$

Since $f(z)$ is analytic, $f'(z)$ must have a unique value in whatever manner $\Delta z \to 0$.

Then From (2) and (3), we get

$$\frac{\partial u}{\partial r} + i\,\frac{\partial v}{\partial r} = \frac{1}{r}\left(-i\,\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial \theta}\right)$$

Equating on both sides real and imaginary parts, we get

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad (4)$$

$$\text{and } \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta} \qquad (5)$$

These equations are the ***Cauchy-Riemann equations*** if $f(z)$ is expressed in polar coordinates.

Note: Differentiating (4) partially with respect to r, we get

$$\frac{\partial^2 u}{\partial r^2} = -\frac{1}{r^2}\frac{\partial v}{\partial \theta} + \frac{1}{r}\frac{\partial^2 v}{\partial r\,\partial \theta} \qquad (6)$$

Differentiating (5) partially with respect to θ, we get

$$\frac{\partial^2 u}{\partial \theta^2} = -r\,\frac{\partial^2 v}{\partial \theta\,\partial r} \qquad (7)$$

Thus using (4), (6) and (7), we get

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0 \quad \left(\text{since} \frac{\partial^2 v}{\partial r\,\partial \theta} = \frac{\partial^2 v}{\partial \theta\,\partial r}\right)$$

EX. 3. Show that $w = f(z) = \bar{z} = x - i\,y$ is not analytic anywhere in the complex plane.

Solution: Let $w = u + i\,v = x - i\,y$.

Here $u = x$ and $v = -y$

Then $\frac{\partial u}{\partial x} = 1, \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0 \textit{ and } \frac{\partial v}{\partial y} = -1$

Hence $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ but $\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

The second of the Cauchy-Riemann equations is satisfied everywhere, but not so the first. So $w = \bar{z}$ is not analytic anywhere in the complex plane.

EX. 4. Show that $w = f(z) = z = x + i\,y$ is analytic anywhere in the complex plane.

Solution: Let $w = u + i\,v = x + i\,y$.

Here $u = x$ and $v = y$

Then $\frac{\partial u}{\partial x} = 1, \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0 \textit{ and } \frac{\partial v}{\partial y} = 1$

Differentiation Formulas: We have already defined the derivative of $w = f(z)$ to be

$$\frac{dw}{dz} = f'(z) = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

This definition is identical in form to that of the derivative of a function of a real variable. Hence the fundamental formulas for differentiation in the domain of complex numbers are the same as those in the case of real variables. Thus we have the following formulas:

(i) If k is a complex constant, then $\frac{d}{dz}(k) = 0$.

(ii) If k is a complex constant and w is a differentiable function, $\frac{d}{dz}(kw) = k\frac{dw}{dz}$.

(iii) If $w_1(z)$ and $w_2(z)$ are two differentiable functions, then $\frac{d}{dz}(w_1 \mp w_2) = \frac{dw_1}{dz} \mp \frac{dw_2}{dz}$.

(iv) $\frac{d}{dz}(w_1 . w_2) = w_1 . \frac{dw_2}{dz} + w_2 . \frac{dw_1}{dz}$

(v) $\frac{d}{dz}\left(\frac{w_1}{w_2}\right) = \frac{w_2 . \frac{dw_1}{dz} - w_1 . \frac{dw_2}{dz}}{w_2^2}$

(vi) If w is a function of $w_1(z)$, $\frac{dw}{dz} = \frac{dw}{dw_1} . \frac{dw_1}{dz}$

(vii) If n is a positive integer, $\frac{d}{dz}(z^n) = n . z^{n-1}$. This can be extended to the case when n is a negative integer or any fraction.

EX. Find where the function $w = f(z) = \frac{1}{z}$ ceases to be analytic.

Solution: Given that $w = f(z) = \frac{1}{z}$

$$\frac{dw}{dz} = \frac{d}{dz}\left(\frac{1}{z}\right) = -\frac{1}{z^2} \; if \; z \neq 0$$

For $z = 0$, $\frac{dw}{dz}$ does not exist. So, w is analytic everywhere except at the point $z = 0$ which is singular point of $f(z)$.

EX. Show that an analytic function with constant real part is constant and an analytic function with constant modulus is also constant.

Solution: Let $w = f(z) = u + i\,v$ be an analytic function.

(a) Let $u(x, y) = a \; constant = c_1$